

Multi-valued dependencies

The definition of multi-valued dependencies is:

Let R be a relation schema and let X and Y be subsets of the attributes of R . Intuitively, the multi-valued dependency $X \twoheadrightarrow Y$ is said to hold over R if in every legal instance r of R , each X value is associated with a set of Y values and this set is independent of the values in the other attributes.

Lets' look at the following:

X	Y	Z
X1	Y1	Z1
X1	Y2	Z1
X1	Y1	Z2
X1	Y2	Z2

In this example $X \twoheadrightarrow Y$ because each X value ($X1$) is associated with a set of Y values ($Y1, Y2$) and this set is independent of the values of Z .

Let's look at the below example. Does this instance of R violate $BC \twoheadrightarrow D$?

A	B	C	D	E
2	2	3	4	5
2	2	3	5	5

BC is equal to $(2,3)$ in both tuples, so for the instance of $(A=2, E=5)$, $BC = (2,3)$ is associated with the set of D values 4 and 5. This is legal.

2nd Example

A	B	C	D	E
2	2	3	4	5
2	2	3	5	5
2	2	3	4	6

In this case BC \rightarrow D is violated because in the instance of (A=2, E=5), BC value of (2,3) is associated with D values of 4 and 5, but in the instance of (A=2, E=6), BC value of (2,3) must be associated with the same set of D values. .

For the above to be legal, there would have to be an additional tuple:

A	B	C	D	E
2	2	3	4	5
2	2	3	5	5
2	2	3	4	6
2	2	3	5	6

3rd example

A	B	C	D	E
2	2	3	4	5
2	2	3	7	5
2	2	3	4	6

This instance violates BC \rightarrow D because the instance of (A=2, E=5), BC value of (2,3) is associated with D values of 4 and 7. The instance of (A=2, E=6), BC value of (2,3) must be associated with the same set. This is addressed below.

A	B	C	D	E
2	2	3	4	5
2	2	3	7	5
2	2	3	4	6
2	2	3	7	6

4th example

A	B	C	D	E
1	2	3	4	5
2	1	3	4	5
1	2	3	6	5
1	2	3	6	6

This example violates $BC \twoheadrightarrow D$ because the first and third tuples establish that the required D values for $(BC = 2,3)$ are 4 and 6 for the corresponding values of A & E. To make this legal, we must add the below:

A	B	C	D	E
1	2	3	4	5
2	1	3	4	5
1	2	3	6	5
1	2	3	6	6
1	2	3	4	6

Now for the case of $(A = 1, E = 5)$, for $(BC = 2,3)$ there are two values of D, 4 and 6. For the case of $(A=1, E=6)$ for $(BC = 2,3)$ there must be two corresponding value of D as well. The case of $(A=2, E=5)$ for $(BC = 1,3)$ is legal because it is a single instance.

5th example

A	B	C	D	E
6	2	3	4	5
6	2	3	6	5
6	2	3	6	6
6	2	3	4	6

$BC \twoheadrightarrow D$ is not violated. For every unique (A,E) there exists a set of D values for each (BC) value.

6th example

From the homework now:

A	B	C
1	2	3
4	2	3
5	3	3
5	3	4

Is $BC \twoheadrightarrow A$ violated? The answer is no. There are no attributes that are not in the multivalued dependency. So the value of (2,3) produces A values of 1 and 4. The value of (3,3) produces an A value of 5. The value of (3,4) produces an A value of 5.

Is $B \twoheadrightarrow C$ violated? The answer is no. For the set of (A=1 and A = 4), B=2 there are set of C values of {3}. In English, this means that for every B value, the same set of C values occurs for every A value. For the set of (A=5), B=3 there are a set of C values of 3 and 4.

If we added the following:

A	B	C
1	2	3
4	2	3
5	3	3
5	3	4
1	2	4

Now we would be in violation, because for the instance of (A=1), B=2, there are 2 C values in the set, 3 and 4. For the instance of (A=4), B=2, there is only 1 C value in the set, 3. An additional tuple of (4,2,4) would have to be added for it to be legal.

7th example

A	B	C	D	E
1	1	1	1	1
1	1	1	2	2
1	2	2	1	1
1	2	2	2	2

In this example, $A \twoheadrightarrow B$ does not hold, $A \twoheadrightarrow C$ does not hold, but $A \twoheadrightarrow BC$ does. For the instance of $(D=1, E=1)$, a value of 1 for A implies the set $\{(1,1), (2,2)\}$ for BC. The instance of $(D=2, E=2)$ reveals the same set for a value of 1 for A.

Another way to look at it.

A *multivalued dependency* (MVD) on R , $X \twoheadrightarrow Y$, says that if two tuples of R agree on all the attributes of X , then their components in Y may be swapped, and the result will be two tuples that are also in the relation.

Employee (Name, Addr, Phones, Emails)

For a multivalued dependency Name \twoheadrightarrow Phones to exist, if the following two tuples occur:

Name	Addr	Phones	Emails
Jack	127 Main	555-1212	jacksmith@xxx.com
Jack	127 Main	493-9999	Jack@yyy.com

Then these tuples must exist as well.

Name	Addr	Phones	Emails
Jack	127 Main	555-1212	jacksmith@xxx.com
Jack	127 Main	493-9999	Jack@yyy.com
Jack	127 Main	493-9999	jacksmith@xxx.com
Jack	127 Main	555-1212	Jack@yyy.com