Problem Solving by Searching

Russell and Norvig, chapter 3.1-3.3
Motivation

- Search plays a key role in many AI systems, e.g.:
  - Route finding (google-maps, internet, airline)
  - VLSI Layout
  - Robot navigation
  - Drug and protein design
  - Video games
Puzzles!
The missionaries and cannibals problem

- **Goal:** transport the missionaries and cannibals to the right bank of the river.
- **Constraints:**
  - Whenever cannibals outnumber missionaries, the missionaries get eaten
  - Boat can hold two people and can’t travel empty
Formulating the problem

A state description that allows us to describe our state and goal:

\[(M_L, C_L, B)\]

- \(M_L\): number of missionaries on left bank
- \(C_L\): number of cannibals on left bank
- \(B\): location of boat - L,R

Initial state: \((3,3,L)\)    Goal: \((0,0,R)\)
Problem solving agents

- Problem formulation
  - States and actions (successor function); initial state
- Goal
  - Desired state of the world.
- Search
  - Determine a sequence of actions that lead to a goal state.
- Execute
  - Perform the actions.
- This is possible because
  - Environment is fully observable and deterministic
  - Agent knows the effects of its actions
Problem solving agents: the graph perspective

- Nodes: all possible states.
- Edges: edge from state $u$ to state $v$ if $v$ is reachable from $u$ by an action of the agent
- Solution: find a path from initial state to goal state
Graph formulation of the missionaries and cannibals problem

- Nodes: all possible states.
- Edges: edge from state $u$ to state $v$ if $v$ is reachable from $u$ (by an action of the agent)

- Problem is now to find a path from $(3,3,L)$ to $(0,0,R)$
Formulating a search problem

- State space $S$ (nodes)
- Successor function: the states you can move to by an action (edge) from the current state
- Initial state
- Goal test
  - is a state $x$ a goal?
- Cost
State: 33L three missionaries and three cannibals on left bank, boat is on left bank
Actions (operators):
CCR - transport two cannibals to the right bank
MCL - transport a missionary and a cannibal to the left bank

Why no MMR transition from this state?
The (partially expanded) search tree

Actions:
CCR - transport two cannibals to the right bank
MCL - transport a missionary and a cannibal to the left bank
Repeated states

The search tree contains duplicate nodes, since the underlying graph in which we are searching is not a tree!
To address this, need to track previously explored states.
Aside: Searching the graph

function GRAPH-SEARCH(problem) returns a solution, or failure
initialize the frontier using the initial state of problem
initialize the explored set to be empty
loop do
  if the frontier is empty then return failure
  choose a leaf node and remove it from the frontier
  if the node contains a goal state then return the corresponding solution
  add the node to the explored set
  expand the chosen node
  add the resulting nodes to the frontier only if not in the frontier or explored

The difference between BFS and DFS is in how the node is chosen and/or how it is added to the frontier: queue versus stack.
Searching the state space

- Often it is not feasible (or too expensive) to build a complete representation of the state graph.
- A problem solver must construct a solution by exploring a small portion of the graph.
Searching the state space
Searching the state space
Searching the state space
Searching the state space
Searching the state space

Search tree
Searching the state space
The 8 puzzle

- States?
- Initial state?
- Actions?
- Goal test?
- Path cost?
The 8 puzzle

- States? Integer location of each tile. How many of them are there?
- Initial state? Any state
- Actions? (tile, direction)
  where direction is one of \{Left, Right, Up, Down\}
- Goal test? Check whether goal configuration is reached
- Path cost? Number of actions to reach goal
- Is the search graph a tree?
(n²-1)-puzzle

\[
\begin{array}{ccc}
8 & 2 & \\
3 & 4 & 7 \\
5 & 1 & 6 \\
\end{array}
\quad
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
13 & 14 & 15 & . . . \\
\end{array}
\]
The 15-puzzle

Sam Loyd offered $1,000 of his own money to the first person who would solve the following problem:
But no one ever won the prize!!
Why?
Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)
Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)
- The cost of a path is the sum of the edge costs along this path
- An optimal solution is a solution path of minimum cost
- There might be no solution!
Path Cost

- An edge cost is a positive number measuring the “cost” of performing the action corresponding to the edge, e.g.:
  - 1 in the 8-puzzle example

- We will assume that for any given problem the cost $c$ of an edge always satisfies:
  \[ c \geq \varepsilon > 0, \text{ where } \varepsilon \text{ is a constant} \]

- Why? Has to do with the cost of arbitrarily long paths
Goal State

- It may be explicitly described:

- or partially described:

- or defined by a condition, e.g., the sum of every row, of every column, and of every diagonal equals 30

("a" stands for “any” other than 1, 5, and 8)
Another example: the 8 queens problem

- Incremental vs. complete state formulation:
  - Incremental formulation starts with an empty state and involves operators that augment the state description
  - A complete state formulation starts with all 8 queens on the board and moves them around
8 queens problem: representation is key

Incremental formulation
- States? Any arrangement of 0 to 8 queens on the board
- Initial state? No queens
- Actions? Add queen in empty square
- Goal test? 8 queens on board and none attacked
- Path cost? None

64 x 63 x … 57 ~ 3 x 10^{14} states to investigate

Is the search graph a tree?
A better representation

Another incremental formulation:

- **States?** n (between 0 and 8) queens on the board, one in each of the n left-most columns; no queens attacking each other.
- **Initial state?** No queens
- **Actions?** Add queen in leftmost empty column such that it does not attack any of the queens already on the board.
- **Goal test?** 8 queens on board

2057 possible sequences to investigate
n-queens problem

- A solution is a goal node, not a path to this node (typical of design problem)
- Number of states in state space:
  - 8-queens $\rightarrow$ 2,057
  - 100-queens $\rightarrow$ $10^{52}$
- But techniques exist to solve n-queens problems efficiently for large values of n
Path planning

What is the state space?
Path planning

This path is the shortest one in the discretized space, but not in the original space.
An alternative formulation

The visibility graph has as its nodes the start and destination points and the vertices of the obstacles.
Cost of a step is the length of the step
An alternative formulation

The solution in this space is the same as in the continuous space!

Lozano-Pérez, Tomás; Wesley, Michael A. (1979), "An algorithm for planning collision-free paths among polyhedral obstacles", *Communications of the ACM* 22 (10): 560–570,
Assumptions in Basic Search

- The world is static
- The world is discretizable
- The world is fully observable
- The actions are deterministic

Search remains an important tool even if not all these assumptions are satisfied