Permissions and Critical Sections
A process holding off on replies is a cue
For the critical section, there’s a queue
You either collect permissions from all
Or from curated subsets that are small
Because messages, these subsets curtail
An added perk is that the system will scale

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Topics covered in this lecture
- Logical clocks
- Distributed Mutual Exclusion
  - Multicast & logical clocks [Agarwala & Ricart]
  - Maekawa’s voting based algorithm
**Logical Clocks**

Logical Clocks: If two processes do not interact with each other

- Their clocks **need not** be synchronized
- Lack of synchronization is not observable
  - Does not cause problems
Lamport’s logical clocks

- The **happens-before** relation

  - If $a$ is event of message sent by one process; and $b$ is event of message being received in another process
    - Then $a \Rightarrow b$ is true

Some more things about the happens-before relation

- If $a \Rightarrow b$ and $b \Rightarrow c$, then $a \Rightarrow c$
  - Transitive

- If events $x$ and $y$ occur in processes that do not exchange messages, then ...
  - $x \Rightarrow y$ is not true
  - But, neither is $y \Rightarrow x$
  - These events are said to be **concurrent**
An example of Lamport’s algorithm:

Each clock runs at a constant (but different rate)
Implementing Lamport’s clocks

1. Before executing an event; $P_i$ executes
   \[ C_i \leftarrow C_i + 1 \]
2. When $P_i$ sends a message $m$ to $P_j$; it sets $m$’s timestamp $ts(m)$ to $C_i$ in previous step
3. Upon receipt of message $m$, $P_j$ adjusts its own local counter
   \[ C_j \leftarrow \text{max}\{C_i, ts(m)\} \]
   do step 1 and deliver message

An application of Lamport’s clock:
User has $1000 in bank account initially

- Add $100 to account
  - San Francisco
  - New York
  - Replicated Database
  - Add $100 ... Total: $1100
  - Give 1% interest on total: $11
  - Balance: $1111

- Update with 1% interest
  - Give 1% interest ... Total: $1010
  - Add $100
  - Balance: $1110
There is a difference when the orders are reversed

- Our objective for now is consistency
- Both copies must be exactly the same

- Situations like this require **totally-ordered multicast**
  - All messages are delivered in the same order to each receiver
  - Lamport’s logical clocks allow us to accomplish this in a completely distributed fashion

Using Lamport’s clock to order messages

- Process puts received messages into local queue
  - Ordered according to the message’s timestamp
- Message can be delivered only if it is **acknowledged** by all the other processes
- If a message is at the head of the queue, and acknowledged by all processes
  - It is delivered and processed
Other types of logical clocks

- Vector clocks
- Matrix clocks

Mutual Exclusion using Multicast & Logical Clocks  
{Ricart & Agarwala's Algorithm}
Requirements for distributed mutual exclusion

- **ME1**: At most one process may execute in the critical section at a time
  - Safety

- **ME2**: Requests to enter and exit the critical section eventually succeed
  - Liveness: Freedom from deadlocks and starvation

- **ME3**: If one request happened-before another, then entry to the CS is granted in that order

Evaluation of the algorithms

- **Bandwidth consumed**: Proportional to number of messages sent in each entry and exit operation

- **Client delay** incurred by process for each entry or exit operation

- Effect on throughput of the system
  - Synchronization delay between one process exiting critical section and next process entering it
  - Throughput is greater when synchronization delay is shorter
Agarwala & Ricart’s algorithm using multicast and logical clocks

- Processes that require entry to a critical section multicast a request message
  - *Enter it only when all other processes have replied* to request

- Process’ replies to a request are designed to ensure that ME1, ME2, and ME3 are met

The setting

- Processes $p_1, p_2, \ldots, p_N$ have distinct identifiers
- Processes have communication channels to each other
- Each process $p_i$ keeps a Lamport clock
- Messages requesting entry are of the form $<T, p_i>$
  - $T$ is the sender’s timestamp and $p_i$ is the sender’s identifier
Each process records its state

- **Released**
  - Outside the critical section

- **Wanted**
  - Wanting entry into the critical section

- **Held**
  - Being in the critical section

Entering the critical section

- If a process requests entry and the state of all other processes is **Released**
  - All processes respond immediately and the entry is granted

- If a process requests entry and some process is in the state **Held**
  - **That holding process will not reply** to requests *until it has finished* with the critical section
  - All other processes respond
Entering the critical section

- If two or more processes request entry at the same time?
  - Request with the lowest timestamp will be first to collect N-1 replies
  - If the Lamport timestamps are the same?
    - Requests are ordered based on their identifiers

- When a process requests entry?
  - Defers all processing requests from other processes until its own request has been sent

Multicast synchronization

Initial Condition:
- \( p_3 \) not interested in entering critical section
- \( p_1 \) and \( p_2 \) request entry concurrently
- Timestamp of \( p_1 \)'s request: 41
- Timestamp of \( p_2 \)'s request: 34

\( p_2 \) enters the critical section
Achieving the properties ME1, ME2 and ME3

- If two processes \( p_i \) and \( p_j \) (\( i \neq j \)) enter critical section at the same time?
  - Both these processes would have replied to each other; but the pairs \( \langle T_i, p_i \rangle \) are totally ordered
    - So it’s impossible
  - Requests to enter and exit the critical section **eventually succeed** because requests are served based on timestamps
    - Satisfies ME2 and ME3 (order)

Evaluation of the algorithm

- Gaining entry takes \( 2(N-1) \) messages
  - \( N-1 \) to multicast the request, followed by \( N-1 \) replies
  - Expensive in terms of bandwidth utilization

- Synchronization delay
  - Just one message transmission time
    - Previous algorithms incurred round-trip delays
Some observations [1/2]

- One of the problems with the central server algorithm was that it was a single point of failure
- Here, the single point of failure has been replaced by N points of failure
  - If any process crashes, it will fail to respond to requests
    - This silence is interpreted (incorrectly) as a denial of permission
    - Blocks ALL subsequent processes from entering the critical section
- Solution: To have timeout mechanisms in place

Some observations [2/2]

- Another problem with the central server algorithm was that making it handle all requests can lead to a bottleneck
- In this setup all processes are involved in all decisions
- Improvements?
  - Getting permission from everyone is an overkill
  - All we need is to prevent two processes from entering the CS at the same time
Maekawa’s voting algorithm for distributed mutual exclusion

Maekawa’s solution to distributed mutual exclusion

- In order for a process to enter a critical section it is not necessary for all peers to grant access
  - Obtain permission from subsets of peers
  - Subsets used by any two peers must overlap

- Candidate process must collect sufficient votes to enter critical section
How mutual exclusion is achieved

- Processes at the intersection of two sets of voters ensure this
  - Cast votes for only one candidate

Voting sets

- There is a voting set $V_i$ associated with each process $p_i$ ($i=1,2,\ldots,N$)

$$V_i \subseteq \{p_1,p_2,\ldots,p_N\}$$
**Voting sets**

- The sets $V_i$ are chosen such that, for all $i, j = 1, 2, ..., N$
  
  $p_i \in V_i$
  
  $V_i \cap V_j \neq \emptyset$
  
  $|V_i| = K$  
  
  To be fair, each process has a voting set of the same size
  
  Each process $p_j$ is contained in $M$ of the voting sets $V_i$

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**The optimal solution to the Maekawa’s algorithm**

$K \sim \sqrt{N}$

$M = K$

Each process is in **as many of the voting sets** as there are **elements in one of the sets**
Calculation of voting sets

- Is not trivial
- As an approximation
  - Place processes in a $\sqrt{N}$ by $\sqrt{N}$ matrix
  - Voting set $V_i$ is the union of the row and column containing $p_i$
  - Voting set size is then $\sim 2\sqrt{N}$

Maekawa’s voting sets

Example

$\begin{align*}
R_1 &= \{1, 2, 3, 4\} \\
R_2 &= \{1, 5, 6, 7\} \\
R_3 &= \{1, 8, 9, 10\} \\
R_4 &= \{1, 11, 12, 13\} \\
R_5 &= \{1, 14, 15, 16, 17\} \\
R_6 &= \{1, 18, 19, 20, 21\}
\end{align*}$

$\begin{align*}
K &= 4 \\
N &= 13
\end{align*}$

$\begin{align*}
R_1 &= \{1, 2, 3, 4, 5\} \\
R_6 &= \{1, 6, 7, 9, 9\} \\
R_{10} &= \{1, 10, 11, 12, 13\} \\
R_{14} &= \{1, 14, 15, 16, 17\} \\
R_{15} &= \{1, 18, 19, 20, 21\}
\end{align*}$

Entering the critical section

- To obtain entry into the critical section, each $p_i$ sends request message to all $K$ members of $V_i$
  - Including itself
- $p_i$ cannot enter critical section till it has received all $K$ reply messages

The reply message

- When a process $p_j$ in $V_i$ receives $p_i$’s request message it sends a reply message immediately unless ...
  - Its state is HELD
  - It has replied (voted) since it last received a release message
The release message

- To leave the critical section, \( p_i \) sends **release message** to all \( K \) members of \( V_i \) (incl. itself)

- When a process receives a release message?
  - **Removes the head** of its queue of outstanding requests and **sends a reply** (vote) in response to it

Satisfying the safety property

- If it were possible for \( p_i \) and \( p_j \) to enter the critical section at the same time, then …
  - Processes in \( V_i \cap V_j \neq \emptyset \) would have voted for both \( p_i \) and \( p_j \)

- But a process can make at **most one vote** between successive receipts of a release message
  - So it is impossible for \( p_i \) and \( p_j \) to both enter the critical section
But the basic algorithm is deadlock prone

- Consider three processes $p_1$, $p_2$, and $p_3$ with $V_1 = \{p_1, p_2\}$, $V_2 = \{p_2, p_3\}$, and $V_3 = \{p_3, p_1\}$.

- If 3 processes concurrently request entry to the critical section it is possible for:
  - $p_1$ to reply to itself and hold-off $p_2$
  - $p_2$ to reply to itself and hold-off $p_3$
  - $p_3$ to reply to itself and hold-off $p_1$
  - Each process receives one of two replies; none can proceed.

Resolving the deadlock issue

- Processes queue requests in the happened-before order.
  - This also allows ME3 to be satisfied besides ME2.
Analyzing the performance of the algorithm

- Bandwidth utilization
  - \(2\sqrt{N}\) messages per entry into the critical section
  - \(\sqrt{N}\) messages per exit
  - Total of \(3\sqrt{N}\) is superior to \(2(N-1)\) required by the previous algorithm (Ricart and Agarwala)
    - If \(N \geq 3\)
- Synchronization delay
  - Round-trip time

The contents of this slide set are based on the following references
