Colorado State University

CS 475 Parallel Programming Wavefront Parallelization

Sanjay Rajopadhye
Colorado State University

Outline

- Parallelizing programs with dependences
  - None of the loops in the program can be parallelized
- Dependence analysis
- Fine grain wavefronts
  - Determining the orientation of wavefronts
  - Transforming the program

Colorado State University 2

Parallelization

- Consider two statement instances x and y, where x executes after y in the sequential version of the program, that we want to parallelize
- x depends on y if x and y access (read or write) the same memory location, notation: y ← x
- Three kinds:
  - Y: write ← X: read RAW: read after write (true)
  - Y: read ← X: write WAR: write after read (anti)
  - Y: write ← X: write WAW: write after write (output)
  - Y: read ← X: read RAR: write after read (input)

Colorado State University 3

Parallelization

- When true, anti, or output dependences occur in a sequential program, their order cannot be changed when parallelizing the program. WHY?
- changing the order changes the outcome of the program

Colorado State University 4

Wavefront Parallelization

- How to parallelize computations (e.g., loops in OpenMP) that have dependences:
  - None of the loop iterations are independent

Simple examples

```
for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        A[i,j] = foo(A[i,j-1], A[i-1,j])

for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        B[j] = bar(B[j-1], B[j])

for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        C[i] = baz(C[i-1], C[i])
```

Iteration Space & Data Space

- Iteration Space: set of values that the loop iterators can take
  - Rectangular region, with "corners" [1,1] and [N-1, M-1]
- Data Space: set of values of array indices accessed by the statements in the program
  - Ex 1: 2-D table, (nearly) identical to the iteration space
  - Ex 2: 1D array, bounded by [0, M-1]
  - Ex 3: 1D array, bounded by [0, N-1]

References and Dependences

- Reference: a occurrence of an array variable on either
  - left hand side (write reference)
  - right hand side (read)

of a statement in the loop body

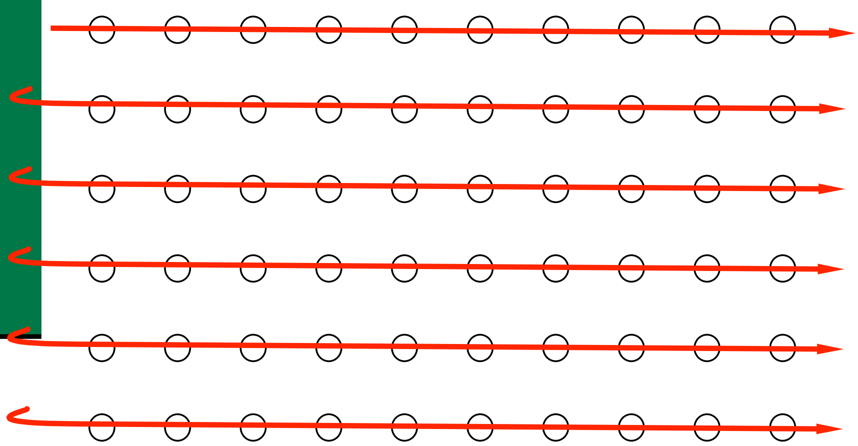
- Dependences: specify which iteration points depend on which others
  - can be refined if/when there are multiple statements in the program

Finding the dependences

- Very hard problem (undecidable in general) but we have simple cases
  - An iteration point [i, j] reads a memory location
  - (Many) iterations (may) have written to that location
  - Find this set (as a function of [i,j])
  - Find the "most recent writer" in this set (again, as a function of [i,j])

Colorado State University 9

Execution order

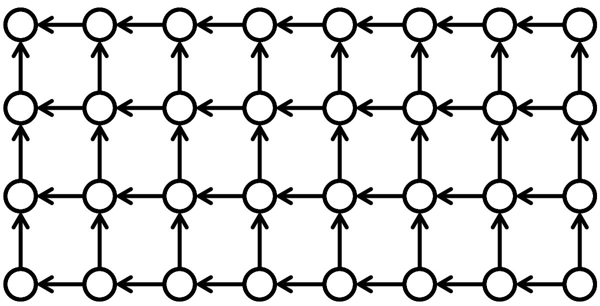


Colorado State University 10

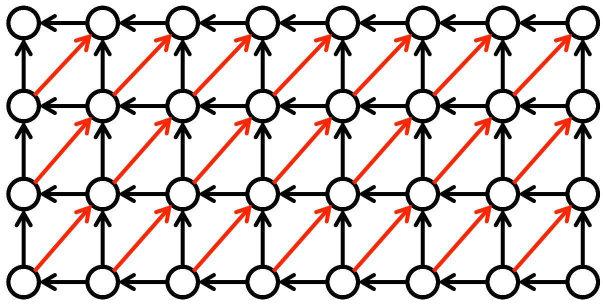
Solutions to examples

- Ex1 and Ex2 (same solution, even though the data space is very different). Iteration [i,j] depends on:
  - [i, j-1] and [i-1, j] neighbors on west and north
  - Ex2 has an additional (memory based dependence)
    - Iteration [i-1, j+1] reads a memory location that the iteration [i, j] is overwriting, that must also happen before [i, j] so it cannot be executed before its northeast neighbor
- Ex3 is more complicated
  - [i,j] depends on [i,j-1] and [i-1, M-1]

Dependence Graph (Ex 1)



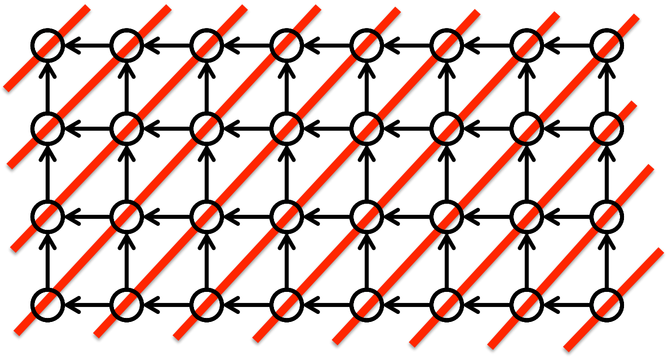
Dependence Graph (Ex 2)



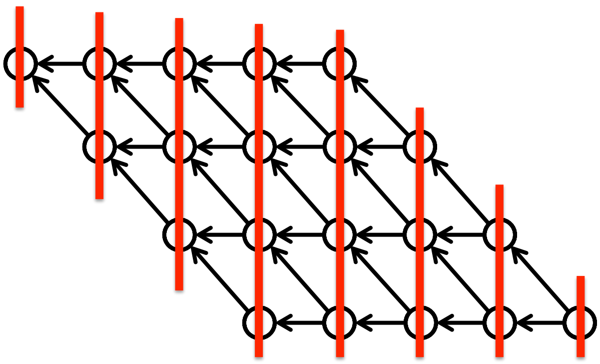
(Finally) the parallelization

- Now that we know the dependences between iterations (the dependence graph)
  - Analyze to determine what can happen at what time (hopefully many things can happen at the same time)
  - Rewrite the program to represent this new order

Ex1 wavefront parallelization



Redraw the graph



Writing the (OpenMP) code

- Node [i, j] is mapped to [p, t]
  - (i, j -> p, t) = (i, j -> i, i+j-1)
- Inverse of the transformation:
  - (p, t -> i, j) = (p, t -> p, t-p+1)
- Determine the transformed iteration space
- Write loops that traverse this
- Outer loop must be the time
- Inner loop is marked to be executed in parallel with

`#pragma omp parallel for`

- Write the new loop body

Control structure

Source → Loop bounds → Iteration space (inequalities)

↓

Target ← Loop bounds ← Transformed Iteration space (inequalities)

Control structure

```
for (i=1; i<N; i++)
    for (j=1; j<M; j++)
```

→ {i, j | 1<=i<=N-1; 1<=j<=M-1}

{p, t | 1<=p<=N-1; 1<=(t-p+1)<=M-1}

{p, t | 1<=p<=N-1; p<=t<=M+p-2}

```
for (t=1; t<=N+M-3; t++)
    for (p=max(1,t-M+2); p<=min(t, N-1); p++) //this is parallel
```


From inequalities to loops

- Work "inside out," i.e., generate bounds on the innermost dimension (say z_n) first
- For each inequality, rearrange it into the form:

  $a_n z_n \geq exp$, for some constant coefficient a_n
  - If a_n is positive, exp/a_n is a lower bound on z_n
  - Otherwise, $-exp/a_n$ is an upper bound
- Let $l_1 \ldots l_m$ be the lower bound expressions and $u_1 \ldots u_{m'}$ be the upper bound expressions.
- The innermost loop is (with one caveat):

  for (z_n = max($l_1 \ldots l_m$); z_n < min($u_1 \ldots u_{m'}$); z_n++)
- Recurse on the outer n-1 dimensions

Recursion: eliminate z_n

- For each pair, u_i, l_j, introduce an inequality, $u_i \geq l_j$
- Let the collection of these inequalities define the iteration space I_{n-1}
  - I_{n-1} is an $(n-1)$-dimensional iteration space (doesn't involve z_n)
  - The first $n-1$ coordinates of every point in the original iteration space, I_n also satisfy I_{n-1}
  - I_n is the intersection of I_{n-1} and loop bounds

Example (on doc cam)

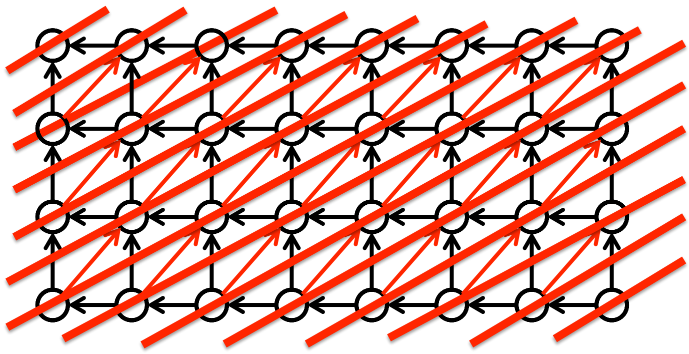
New loop body

- At each point [t, p] in the new loop,
  - Determine the original iteration point that was mapped to [t, p] (*inverse* of the rectangle-to-parallelogram transformation) Given [t, p] = [i+j-1, j] solve for [i, j] in terms of t and p.
  - Add synchronization (optional)
  - Optionally, change memory

New loop body

```
int i, j;
    for (t=1; t<=N+M-3; t++)
    #pragma omp parallel for private i, j
    for (p=max(1,t-M+2); t<=min(t,N-1); p++) {
       i = p;
       j = t-p+1;
       // insert old loop body (unchanged) here:
       // we chose t and p as brand new index names
       A[i,j] = foo(A[i,j-1], A[i-1,j]);
       }
```

Ex2 wavefront parallelization



Example 2 (contd)

- Mapping is (i, j -> p, t) = (i, j -> i, 2i+j-2)
- Inverse of the transformation:
  - (p, t -> i, j) = (p, t -> p, t-2p+2)
- Transformed iteration space:

  {p, t | 1<=p<=N-1; 1<=(t-2p+2)<=M-1}
- Rewrite as:

  {p, t | 1<=p<=N-1; t-M+3<=2p<=t+1}

Write the new loop

Example 2 (contd)

```
for (t=3; t<= 2N+M-5; t++)
    for (p = max(1, ⌈(t-M+3)/2⌉ ); p <= min( ⌊(t+1)/2⌋ , N-1) {
        //NEW LOOP BODY:
        i = p; j = t-2p+2;
        // copy the old body
        B[j]=bar(B[j], B[j-1]);
}
```

$$p = \max\left(1, \left\lceil \frac{t-M+3}{2} \right\rceil\right); \quad p \le \min\left(\left\lfloor \frac{t+1}{2} \right\rfloor, N-1\right)$$

Better way

- Early preoccupation with memory:
  - Memory allocation of the original program is hurting us
- First parallelize the "full table version"
- Then make it use less memory

Ex1 revisited = Ex 2

```
int i, j;
    for (t=1; t<=N+M-3; t++)
    #pragma omp parallel for private i, j
    for (p=max(1,t-M+1); t<=min(t,N-1); p++) {
       i = p;
       j = t-p+1;
       // A[i,j] = foo(A[i,j-1], A[i-1,j]);
       A[i%2, j] = bar(A[i%2, j-1], A[(i-1)%2, j]);
       if (p==N-1) B[j] = A[i%2, j];
}
```


Conclusions

- Only simple (fine-grain wavefronts)
- Not dealing with memory
- Granularity of synchronization/fork-join overhead
- Just the beginning
  - Tiling
  - Tiling + parallelism
  - Memory (remapping)
- Advanced topics in CS 560/575