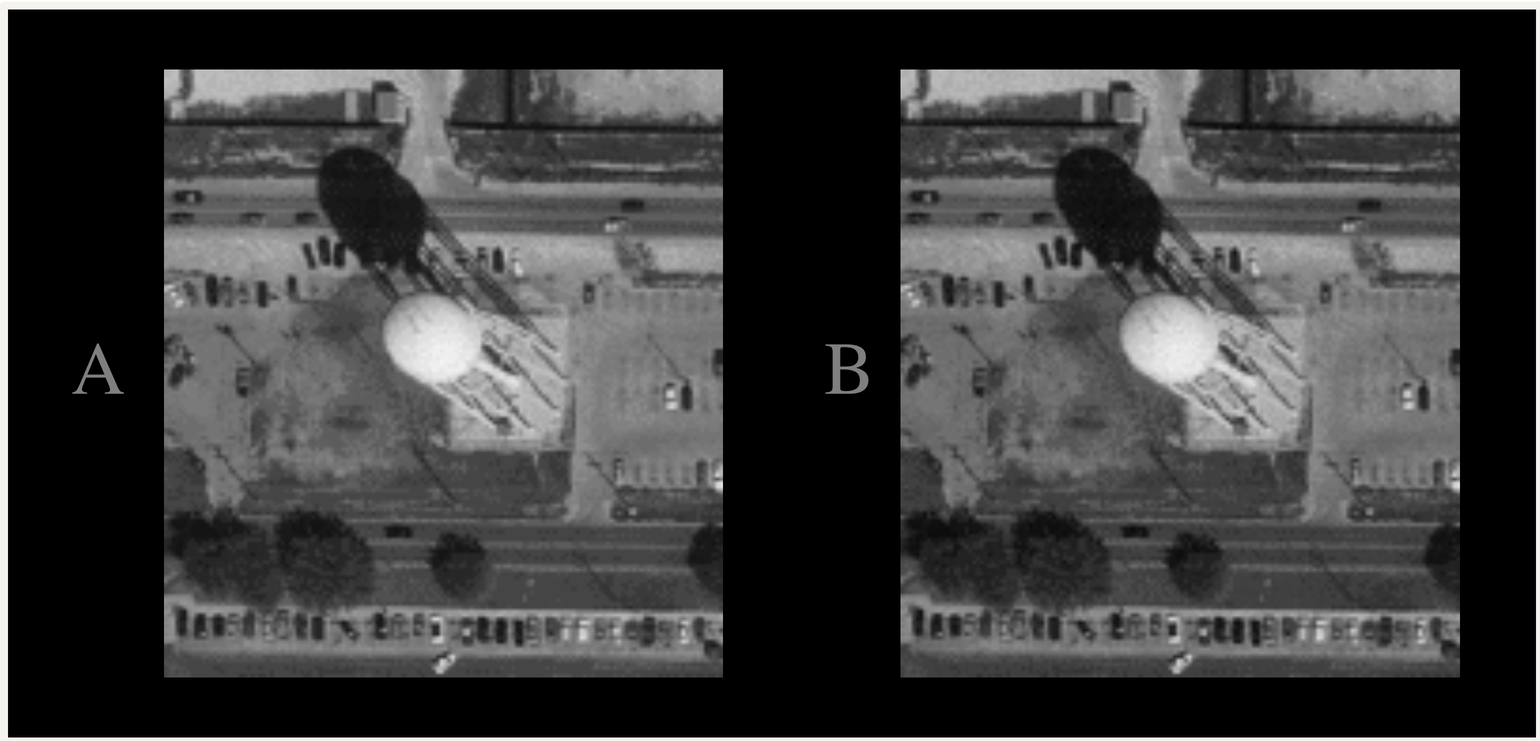
Image Matching

Lecture #04

February 1, 2013

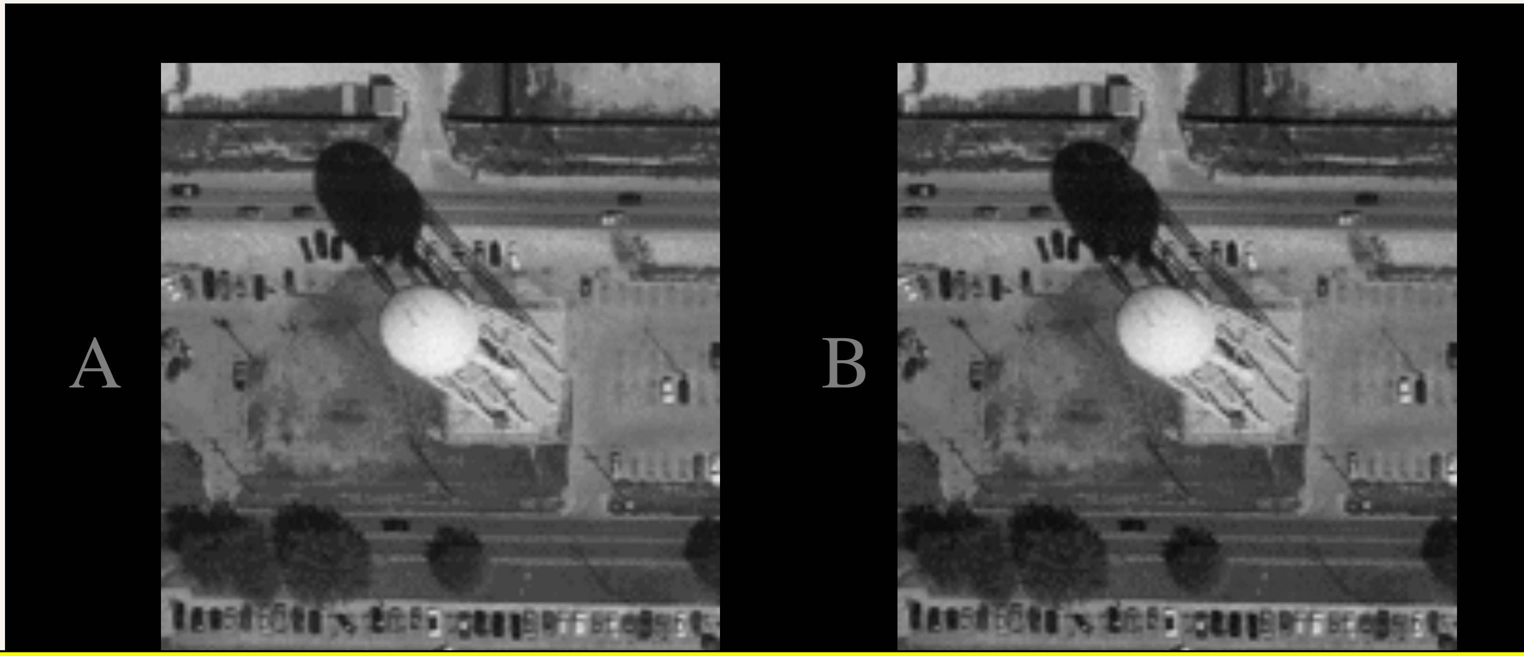
Colorado State University

How do we (directly) compare two images?



Are these images the same? Are they similar?

Pixel-wise Comparison



Or, normalized by image area, about 5 grey values per pixel.

$$8{,}140 = \sum_{x}^{x<148} \sum_{y}^{y<161} \left| A(x,y) - B(x,y) \right|$$

Backup - what is "similarity"?

Consider two vectors/points.

$$X = \begin{vmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{vmatrix} \quad Y = \begin{vmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{vmatrix}$$

Distance vs. similarity:

$$S : R^n \times R^n \rightarrow R$$

$$D : R^n \times R^n \rightarrow R$$

$$S \propto {}^{1}\!/_{D}$$

Common Approaches

- Euclidean (L2)Distance
- City Block (L1) Distance
- Pearson's Correlation
- Linear Correlation

Important, Less Common

- Mahalanobis Distance
- Mutual Information

Simple Distances (norms)

L1 - City Block Distance $$\sum_{x,y}\left(\left|A(x,y)-B(x,y)\right|\right)$$

L2 - Euclidean Distance $$\sqrt{\sum_{x,y}\left(A[x,y]-B[x,y]\right)^2}$$

Generalized L-norm $$\sqrt[l]{\sum_{x,y}\left(\left|A(x,y)-B(x,y)\right|\right)^l}$$

Properties of L1 Distance

Consider the following problem:

Find the unique point "closest" to k other points.

For simplicity, do this in R (a line) with k = 2.

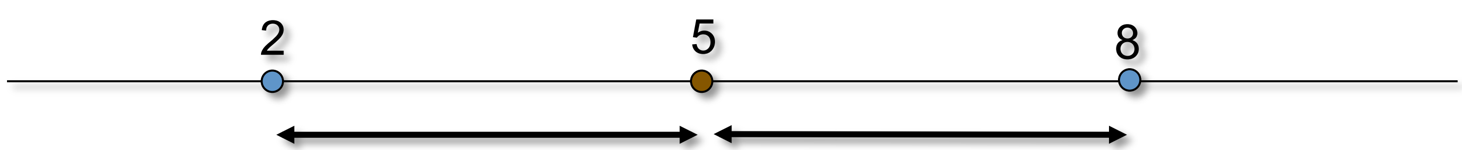
See the problem yet?

$$|2-3|+|8-3|=6 \qquad |2-4|+|8-4|=6$$

In Comparison, Consider L2

Find the unique point "closest" to k other points.

Using L2,



$$\sqrt{(2-5)^2+(8-5)^2} = \sqrt{18} \qquad \sqrt{(2-4)^2+(8-4)^2} = \sqrt{20}$$

Best | Not as Good

Sources of Image Variation

info

- Change in scene content
- Out-of-plane rotation, scale change
- In-plane translation, rotation

- Change in illumination

noise

- Change in mixed-pixels
- Change in gain, f-stop
- Electronic noise

Motivating Pearson's Correlation

Let's play a Game

Two vectors are similar to the extent that the value at a dimension in one lets you predict the value in the other

CS 510 Guess The Next Number Game

Solutions

	Game 1				Game 2			
	X	Y	Y-True	Check	X	Y	Y-True	Check
1	4	3			4	1		
2	1	4			1	3		
3	3	2			3	3		
4	2	5			2	3		
5	5	5			5	0		
6	1	2			1	3		
7	4	5			4	1		
8	5	4			5	1		
9	5	2			5	0		
10	4	3			4	2		
11	4				4			
12	3				3			
13	5				5			
14	3				3			
15	4				4			
16	2				2			
17	1				1			
18	2				2			
19	1				1			
20	1				1			

The results of the Game

- First Game - Random
  - Expected ~20% correct
- Second Game
  - Invert 1-5 to 4-0
  - Add some noise
- Game 2 - features
  - Nearly perfect prediction
  - … to within one value.
- Punch line
  - **Correlation measures predictability!**

CS 510 Guess The Next Number Game

Solutions

	Game 1				Game 2			
	X	Y	Y-True	Check	X	Y	Y-True	Check
1	4	3	3		4	1	1	
2	1	4	4		1	3	3	
3	3	2	2		3	3	3	
4	2	5	5		2	3	3	
5	5	5	5		5	0	0	
6	1	2	2		1	3	3	
7	4	5	5		4	1	1	
8	5	4	4		5	1	1	
9	5	2	2		5	0	0	
10	4	3	3		4	2	2	
11	4		5		4		1	
12	3		3		3		2	
13	5		5		5		1	
14	3		3		3		2	
15	4		4		4		1	
16	2		2		2		4	
17	1		4		1		3	
18	2		5		2		4	
19	1		4		1		4	
20	1		2		1		3	
			Tally				Tally	

Pearson's Correlation

$$\frac{\sum_{x,y}\left(A(x,y)-\overline{A}\right)\left(B(x,y)-\overline{B}\right)}{\sqrt{\sum_{x,y}\left(A(x,y)-\overline{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\overline{B}\right)^2}}$$

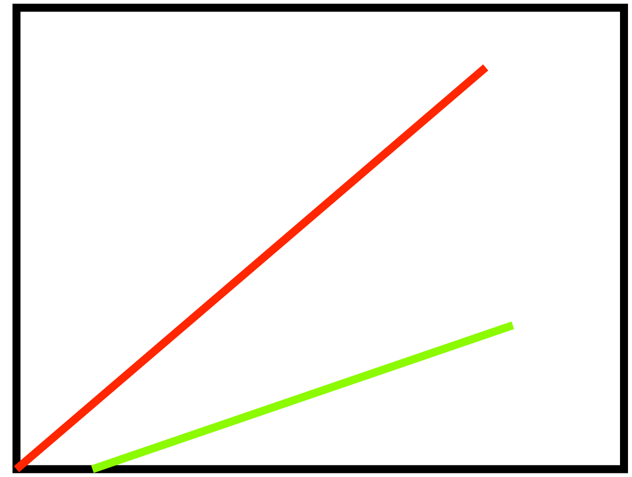
What is the underlying model?

Assumptions of Correlation

$$\frac{\sum_{x,y}\left(A(x,y)-\overline{A}\right)\left(B(x,y)-\overline{B}\right)}{\sqrt{\sum_{x,y}\left(A(x,y)-\overline{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\overline{B}\right)^2}}$$

- Two signals vary linearly
  - Constant shift to either signal has no effect.
  - Increased amplitude has no effect.
- This minimizes sensitivity to:
  - changes in (overall) illumination
  - offset or gain.

Special Cases



- Any two linear functions with positive slope have correlation 1.
  – Only the sign of the slope matters.
- Any two linear functions with differently signed slopes have correlation -1.
  – This is called anti-correlation
  – Anti-correlation = correlation for prediction.
  – For matching, it may or may not be as good…
- Correlation undefined for slope = 0 (σ=0)

Correlation (cont.)

- Correlation is sensitive to:
  - Translation
  - Rotation: in-plane and out-of-plane
  - Scale
- Because it …
  - Assumes pixels align one atop the other.
  - Compares two images pixel by pixel.
- Translation handled by convolution
  - Example, alignment by template matching

Computing Correlation

- Note that adding a constant to a signal does not change its correlation to any other signal, so
  - Let's subtract average A from A(x,y)
  - Let's subtract average B from B(x,y)
  - The mean of both signals is now zero
  - Then correlation reduces to:

$$\frac{A \cdot B}{\sqrt{\sum_{x,y}\left(A(x,y)-\bar{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\bar{B}\right)^2}}$$

Computing Correlation (II)

- For zero-mean signals, we can scale them without changing their correlation scores
  - Multiply A by the inverse of its length
  - Multiply B by the inverse of its length
  - Both signals are now unit length
  - Then correlation reduces to:

$$A \cdot B$$

- Gives rise to 'Correlation Space'.

Correlation Space

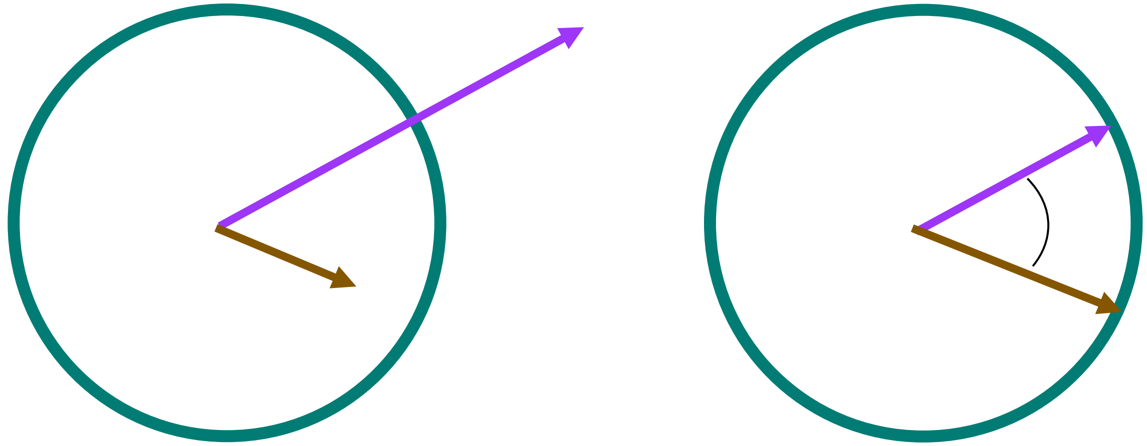
- Why zero-mean & unit-length your images?
- Consider database retrieval
  - Compare new image A …
  - with many images in database.
  - When database images are stored in their zero-mean & unit-length form, then
  - Preprocess A (zero-mean, unit-length)
  - Compute dot products

Correlation Space (II)

- New idea: image as a point in an N dimensional space
    - N = width x height
- Zero-mean & unit-length images lie on an N-1 dimensional “correlation space” where the dot product equals correlation.
    - This is a highly non-linear projection.
    - Points lie on an N-1 surface within the original N dimensional space.
- So consider points in 3-D ….

Correlation Space (II)

- Subtracting mean - translation.
- Length one - project onto sphere.
- Correlation is then:
  - Cosine of angle between vectors (points).



Useful Connection …

- Euclidean distance is inversely proportional to correlation in correlation space.

$$\sqrt{\sum_{x,y}\left(A[x,y]-B[x,y]\right)^2} = \sqrt{\sum_{x,y}A[x,y]^2+\sum_{x,y}B[x,y]^2-2A[x,y]B[x,y]}$$

$$= \sqrt{1+1-2\sum_{x,y}A[x,y]B[x,y]}$$

$$= \sqrt{2-2A\cdot B}$$

$$= \sqrt{2-2Corr(A,B)}$$

➢ Nearest-neighbor classifiers in correlation space maximize correlation