Getting Beyond Distance: (Stats) Cross-Correlation & (EE) Correlation

CS 510
Lecture #9
February 18, 2015

Pearson’s Correlation

Recall from the previous lecture ...

$$\sum_{x,y} (A(x,y) - \bar{A})(B(x,y) - \bar{B})$$

$$\sqrt{\sum_{x,y} (A(x,y) - \bar{A})^2} \sqrt{\sum_{x,y} (B(x,y) - \bar{B})^2}$$

This is a very important equation...
Assumptions of Correlation

• Two signals vary linearly
  – Constant shift to either signal has no effect.
  – Increased amplitude has no effect.
• This minimizes sensitivity to:
  – changes in (overall) illumination
  – offset or gain.

Computing Correlation

• A constant added to A does not change its correlation to any other signal, so
  – Let’s subtract average A from A()
  – Let’s subtract average B from B()
  – The mean of both signals is now zero
  – Then correlation reduces to:

\[
\frac{\sum_{x,y} (A(x,y) - \bar{A})(B(x,y) - \bar{B})}{\sqrt{\sum_{x,y} (A(x,y) - \bar{A})^2} \sqrt{\sum_{x,y} (B(x,y) - \bar{B})^2}}
\]

Know and love the dot product.
Computing Correlation (II)

- For zero-mean signals, we can scale them without changing their correlation scores
  - Multiply A by the inverse of its length
  - Multiply B by the inverse of its length
  - Both signals are now unit length
  - Then correlation reduces to:

\[ A \cdot B \]

- Gives rise to ‘Correlation Space’.

Correlation Space

- Why zero-mean & unit-length your images?
- Consider, for example, database retrieval
  - Compare new image A …
  - with many images in database.
  - When database images are stored in their zero-mean & unit-length form, then
  - Preprocess A (zero-mean, unit-length)
  - Compute dot products
Correlation Space (II)

• New idea: image as a point in an N dimensional space
  • N = width x height
• Zero-mean & unit-length images lie on an N-1 dimensional “correlation space” where the dot product equals correlation.
  – This is a highly non-linear projection.
  – Points lie on an N-1 surface within the original N dimensional space.
• So consider points in 3-D …

Correlation Space (II)

• Subtracting mean - translation.
• Length one - project onto sphere.
• Correlation is then:
  – Cosine of angle between vectors (points).
Useful?

- Yes
- Very commonly used.
- For example
  - Face Rec.
- Google
  - 279,000 hits

Useful Connection ...

- Euclidean distance is inversely proportional to correlation in correlation space.
  \[
  \sqrt{\sum_{x,y} (A[x,y] - B[x,y])^2} = \sqrt{\sum_{x,y} A[x,y]^2 + \sum_{x,y} B[x,y]^2 - 2A[x,y]B[x,y]}
  \]
  \[
  = \sqrt{1 + 2\sum_{x,y} A[x,y]B[x,y]}
  \]
  \[
  = \sqrt{2 - 2A \cdot B}
  \]
  \[
  = \sqrt{2 - 2\text{Corr}(A,B)}
  \]
- Nearest-neighbor classifiers in correlation space maximize correlation / minimized L₂ norm
Limitations

• To match images this way, they must be
  – The same width & height
  – In correspondence: coordinates match

• More importantly, objects in the scene must
  – Be in the same location
  – Be at the same scale
  – Be at the same orientation
  – Be seen from the same viewpoint

New Goal: Find a small image within a larger one

• The image above is a small piece of the image to the right. But from where?
Brute-Force Translation Invariance

To find a small image in a large one, “slide” the small one across the large, computing Pearson’s correlation at every possible position.

Statistical Cross-Correlation

- The process of “slide & correlate” is called cross-correlation
- Complexity is $O(nm)$
  - $N = \#$ of pixels in image ($w \times h$)
  - $M = \#$ of pixels in the template ($w \times h$)
- Highly parallel (every position can be computed independently)
- Still sensitive to
  - Rotation
    - in-plane
    - out-of-plane
  - Scale
  - Perspective
Computing Cross-Correlation

• In cross-correlation, the mask is correlated repeatedly to image windows
  – zero-mean & unit length the mask
  – zero-mean & unit length the image
  – compute the sliding dot product

This is *almost* convolving the image with the mask

Naming conventions

• In Engineering, convolving a normalized mask with the source image is called correlation
  – Is this exactly the same as Pearson’s correlation?
  – Why or why not?
• This is the most common definition of correlation in image processing texts
Application: Tracking

- Cut picture of a target from the first frame of a video
  - Use it as a template /mask
- Correlate the target in the following frames
  - Move to highest correlation peak and cut new target.

Add Scale Correction

ECE 695 Project: OBJECT TRACKING IN VIDEOS, Arun Anbumani, December 18, 2013
Application: Mosaicing

- Take several, overlapping images from a translating camera
  - Camera cannot move along optical axis
- Correlate the whole images to each other
  - Find location where they match the best
  - Stitch them into a single, larger image
Application: Detection

• Could you use correlation to find…
  – Human faces?
  – heads?
  – Hands?
  – bodies?

• What (if anything) might lead to failures?