PCA to Eigenfaces

CS 510

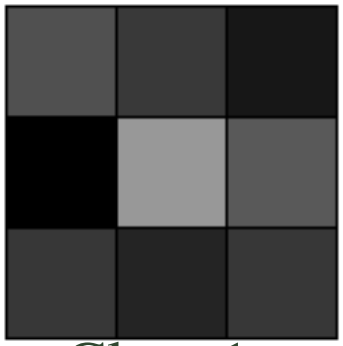
Lecture #16

February 28, 2018

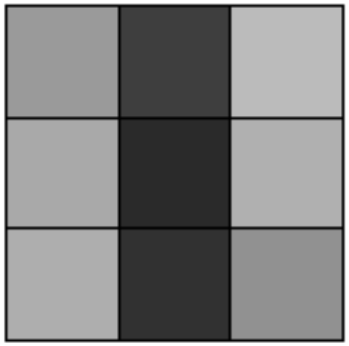
Colorado State University

A 9 dimensional PCA example

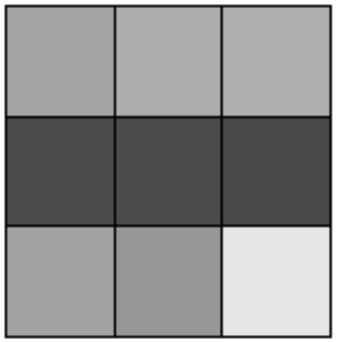
- Class 1 is dark around the edges and bright in the middle.
- Class 2 is light with dark vertical bars.
- Class 3 is light with dark horizontal bars.
- All classes initially use 2 for low value, 7 for high value.
- Each instance is corrupted by sigma=1 Guassian Noise.



Class 1



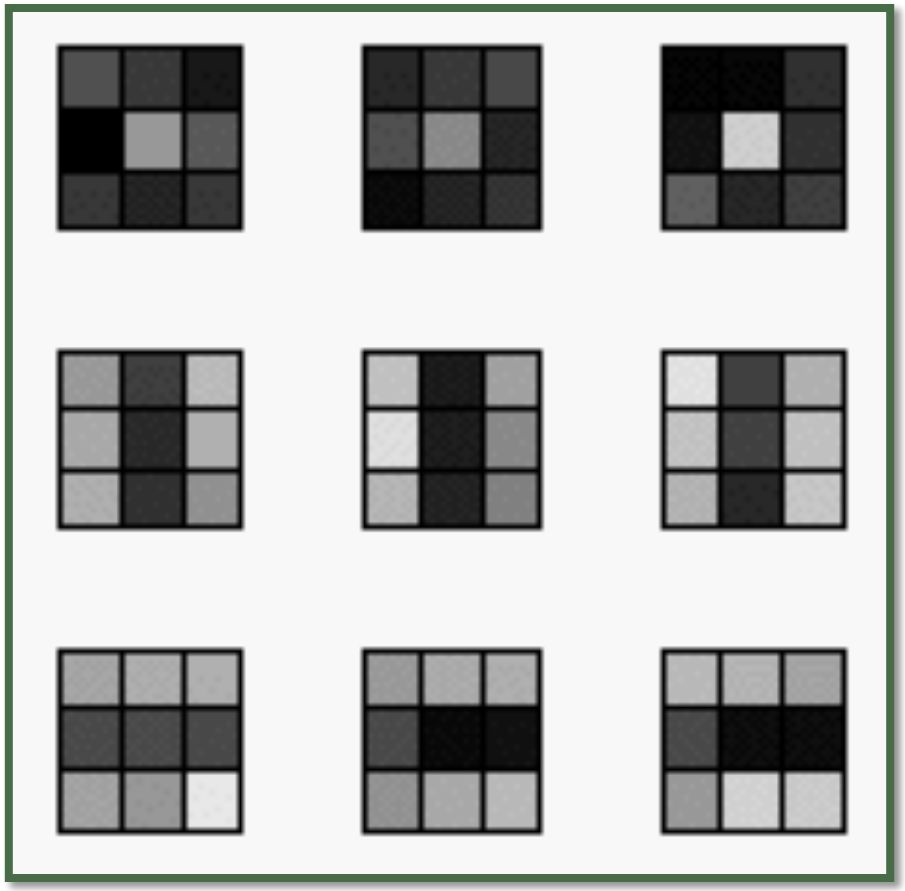
Class 2



Class 3

Eigenspace Example 1

- Consider 3 examples from the 3 classes.



Class 1

Class 2

Class 3

The Image Matrices

- Here they are as matrices.

$$\left[\begin{bmatrix} 1.65 & 3.11 & 2.25 \\ 3.22 & 5.79 & 3.09 \\ 1.10 & 2.47 & 2.96 \end{bmatrix}, \begin{bmatrix} 1.55 & 3.29 & 1.62 \\ 2.91 & 3.88 & .71 \\ 2.35 & 3.60 & 2.46 \end{bmatrix}, \begin{bmatrix} .80 & 2.43 & 2.04 \\ 1.59 & 8.17 & .79 \\ .69 & 1.96 & 4.34 \end{bmatrix},\right.$$

$$\begin{bmatrix} 6.36 & 2.39 & 9.36 \\ 6.05 & .55 & 6.60 \\ 5.97 & 3.49 & 7.33 \end{bmatrix}, \begin{bmatrix} 6.43 & 1.43 & 7.01 \\ 7.66 & 3.20 & 6.66 \\ 6.96 & 1.82 & 7.52 \end{bmatrix}, \begin{bmatrix} 6.52 & .89 & 7.74 \\ 4.80 & 1.97 & 7.58 \\ 5.75 & 1.06 & 7.24 \end{bmatrix},$$

$$\left.\begin{bmatrix} 8.11 & 8.94 & 5.85 \\ 2.63 & 2.60 & 5.16 \\ 7.20 & 6.09 & 6.12 \end{bmatrix}, \begin{bmatrix} 6.94 & 6.68 & 5.99 \\ 3.63 & 3.15 & 1.37 \\ 8.50 & 6.89 & 6.49 \end{bmatrix}, \begin{bmatrix} 7.02 & 7.73 & 7.08 \\ 2.75 & 2.10 & 1.91 \\ 5.92 & 6.85 & 7.16 \end{bmatrix}\right]$$

Normalized Image Vectors

- Each as a 9x1 vector, an unrolled image.
- Each has zero mean and unit length.

$$X = \left[\begin{bmatrix} -.133 \\ .0500 \\ -.102 \\ .0750 \\ .324 \\ .0910 \\ -.188 \\ .00100 \\ -.0630 \end{bmatrix}, \begin{bmatrix} -.117 \\ .127 \\ -.141 \\ .0930 \\ .186 \\ -.152 \\ -.0140 \\ .185 \\ -.0740 \end{bmatrix}, \begin{bmatrix} -.231 \\ -.0450 \\ -.143 \\ -.114 \\ .505 \\ -.163 \\ -.239 \\ -.0710 \\ .0450 \end{bmatrix}, \begin{bmatrix} .0480 \\ -.149 \\ .184 \\ .0710 \\ -.266 \\ .131 \\ .0300 \\ -.0670 \\ .0320 \end{bmatrix}, \begin{bmatrix} .0530 \\ -.202 \\ .0520 \\ .162 \\ -.116 \\ .135 \\ .0860 \\ -.161 \\ .0440 \end{bmatrix}, \begin{bmatrix} .0840 \\ -.229 \\ .124 \\ .0200 \\ -.178 \\ .217 \\ .0410 \\ -.200 \\ .0570 \end{bmatrix}, \begin{bmatrix} .126 \\ .197 \\ -.0290 \\ -.129 \\ -.157 \\ .0370 \\ .0800 \\ .0630 \\ -.0520 \end{bmatrix}, \begin{bmatrix} .0810 \\ .0930 \\ -.00600 \\ -.0660 \\ -.120 \\ -.163 \\ .172 \\ .124 \\ -.0150 \end{bmatrix}, \begin{bmatrix} .0900 \\ .157 \\ .0600 \\ -.113 \\ -.177 \\ -.132 \\ .0310 \\ .126 \\ .0270 \end{bmatrix} \right]$$

Singular Value Decomposition

- Actual values for this example.
- The Eigenvalues are 1.2, 0.55, 0.19 etc.
- The Eigenvectors are columns of U matrix.

$$\begin{bmatrix} -.29 & -.080 & -.31 & -.51 & .40 & .30 & -.17 & -.15 & -.51 \\ -.14 & .46 & .10 & -.48 & -.27 & -.17 & -.46 & -.30 & .35 \\ -.23 & -.060 & .34 & .050 & -.49 & -.29 & -.18 & .16 & -.66 \\ -.23 & -.52 & .64 & -.060 & .12 & .16 & .050 & -.44 & .17 \\ .84 & .040 & .16 & -.10 & -.090 & .21 & -.12 & -.32 & -.30 \\ .080 & -.47 & -.42 & -.36 & -.45 & -.29 & .35 & -.22 & .080 \\ -.14 & -.10 & -.38 & .60 & -.080 & -.020 & -.40 & -.54 & -.050 \\ -.14 & .50 & .080 & .080 & .13 & -.21 & .62 & -.47 & -.23 \\ -.20 & .16 & -.050 & .050 & -.53 & .77 & .21 & 0 & .040 \end{bmatrix} \begin{bmatrix} 1.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .55 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .19 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .030 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .010 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -.29 & -.14 & -.23 & -.23 & .84 & .080 & -.14 & -.14 & -.20 \\ -.080 & .46 & -.060 & -.52 & .040 & -.47 & -.10 & .50 & .16 \\ -.31 & .10 & .34 & .64 & .16 & -.42 & -.38 & .080 & -.050 \\ -.51 & -.48 & .050 & -.060 & -.10 & -.36 & .60 & .080 & .050 \\ .40 & -.27 & -.49 & .12 & -.090 & -.45 & -.080 & .13 & -.53 \\ .30 & -.17 & -.29 & .16 & .21 & -.29 & -.020 & -.21 & .77 \\ -.17 & -.46 & -.18 & .050 & -.12 & .35 & -.40 & .62 & .21 \\ -.15 & -.30 & .16 & -.44 & -.32 & -.22 & -.54 & -.47 & 0 \\ -.51 & .35 & -.66 & .17 & -.30 & .080 & -.050 & -.23 & .040 \end{bmatrix}$$

$$U \qquad D \qquad U^T$$

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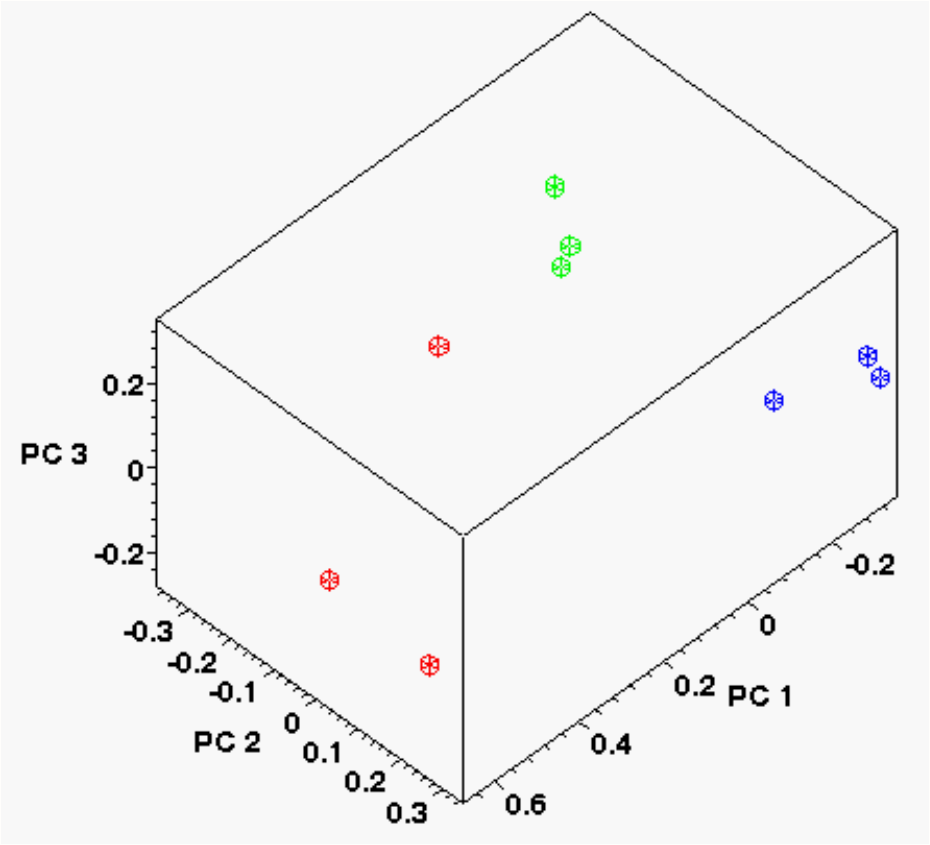
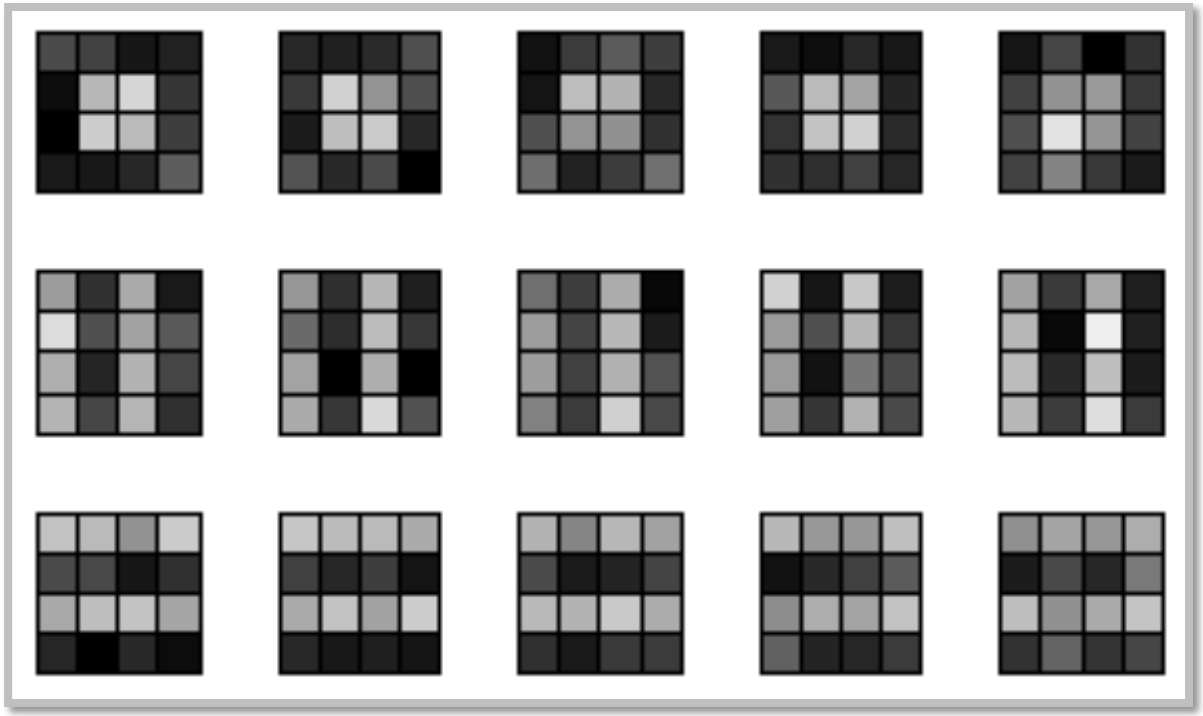
Subspace Projection Pictures



Legend
Class 1
Class 2
Class 3

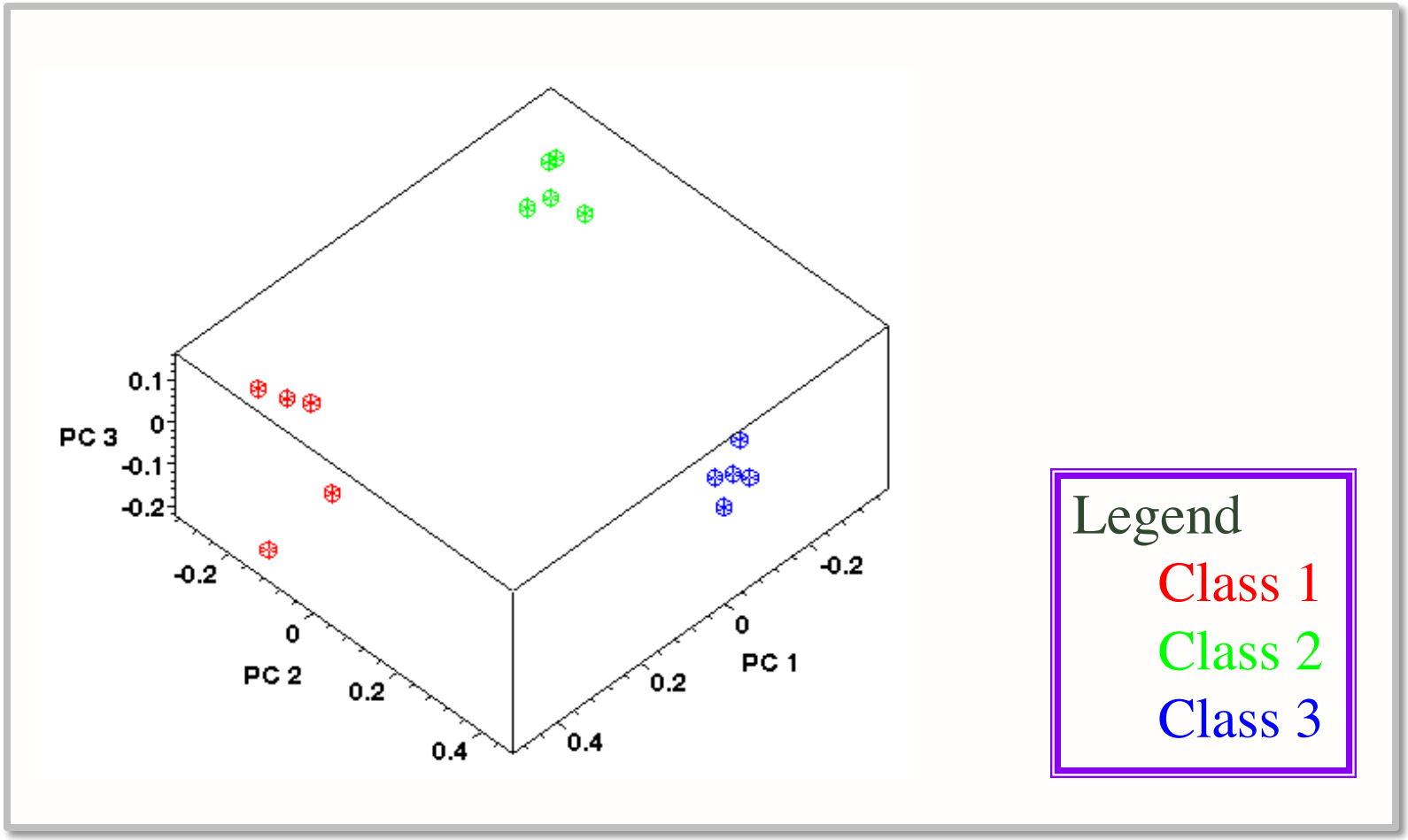
Eigenspace Example 2

- Consider 12 4x4 images.

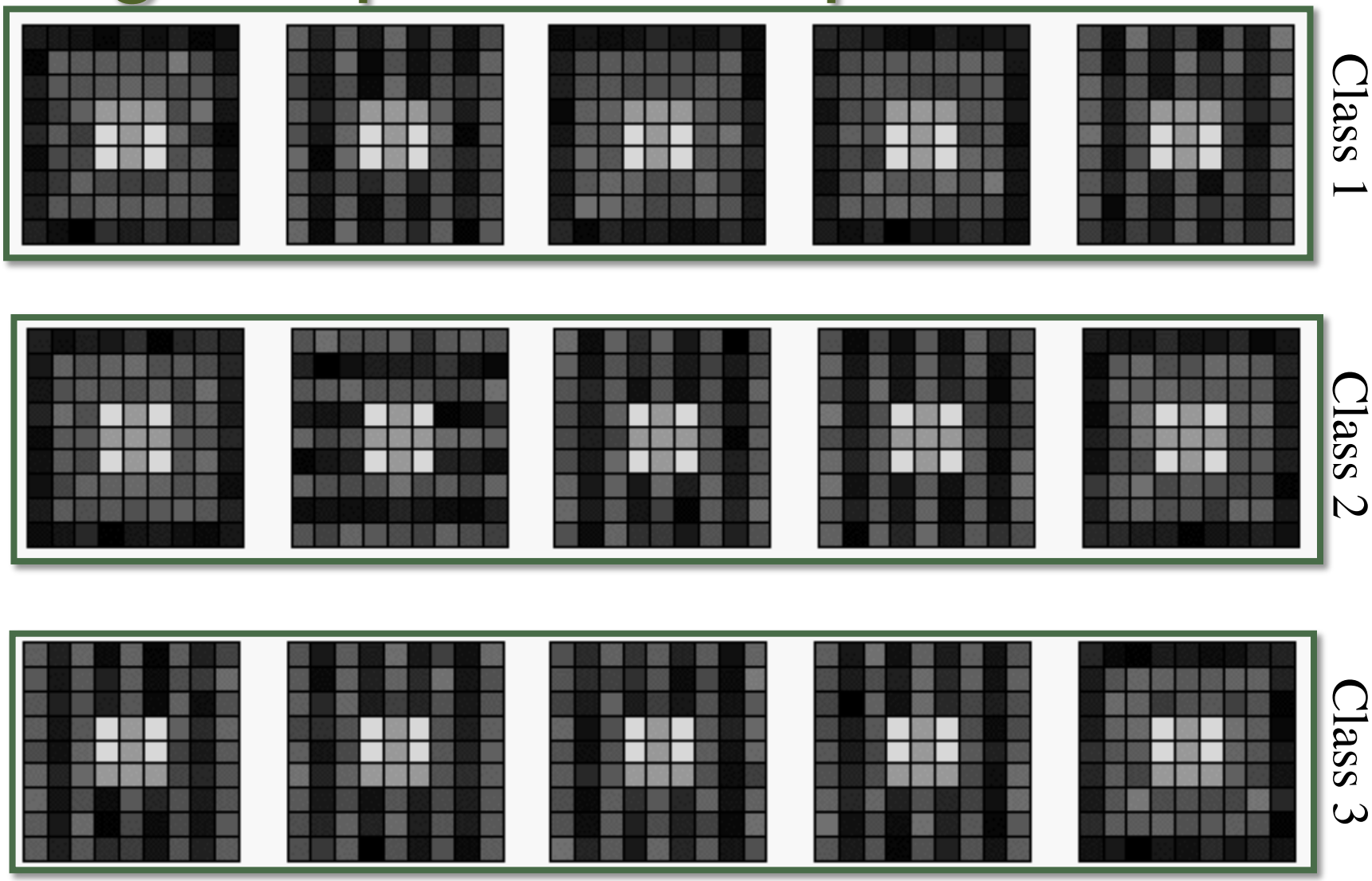


➢ Low value is 2, high is 7, noise sigma 1.0

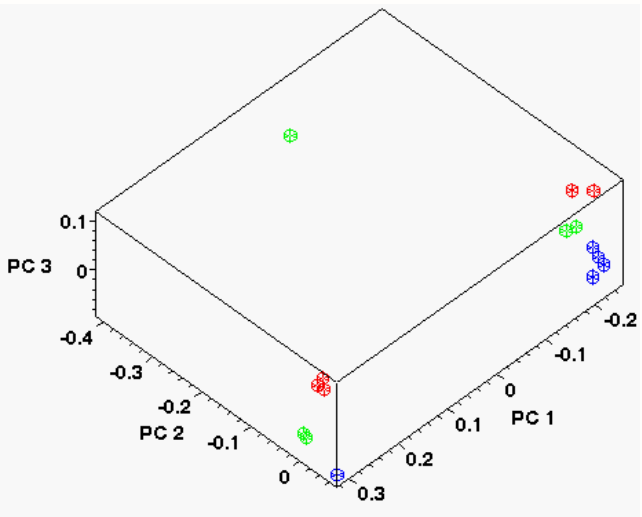
Example 2 Subspace 3D

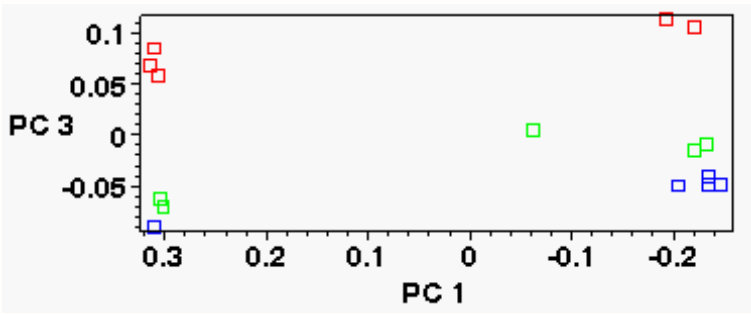


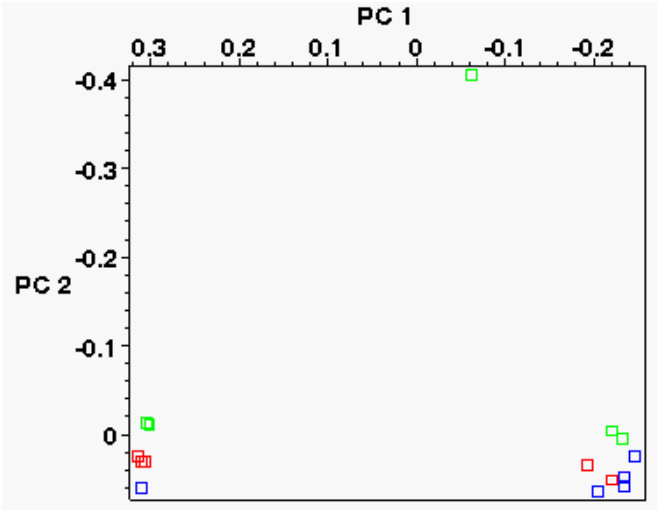
Eigen Space Example 3



Example 3 Subspace







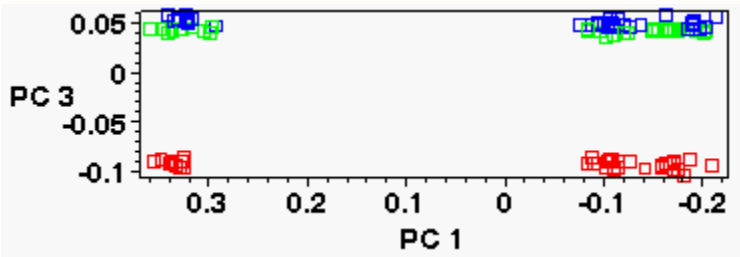
Legend

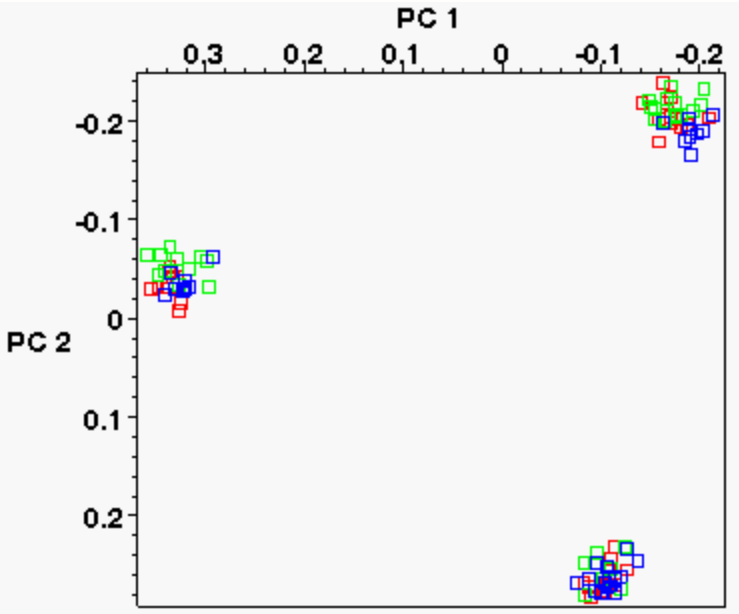
Class 1

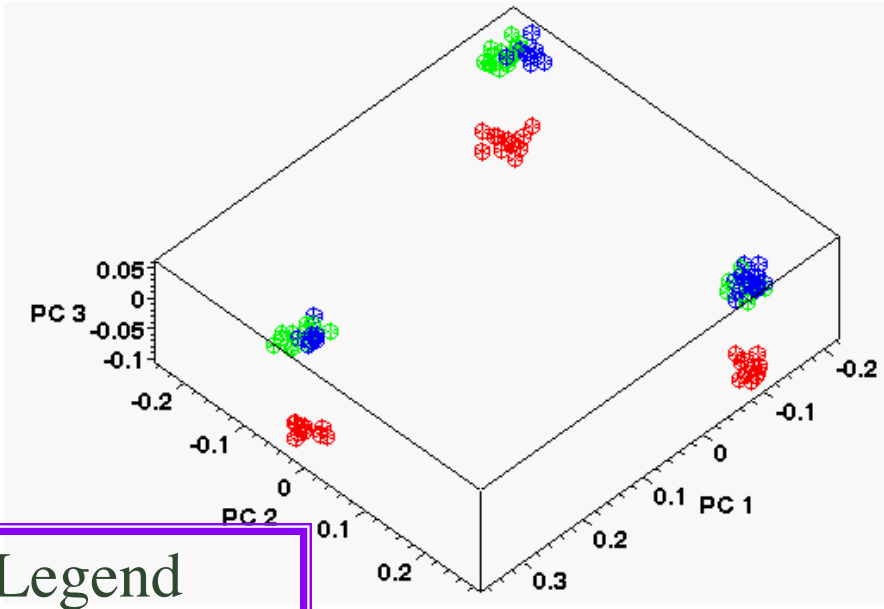
Class 2

Class 3

Example 3 Subspace: 99 Samples







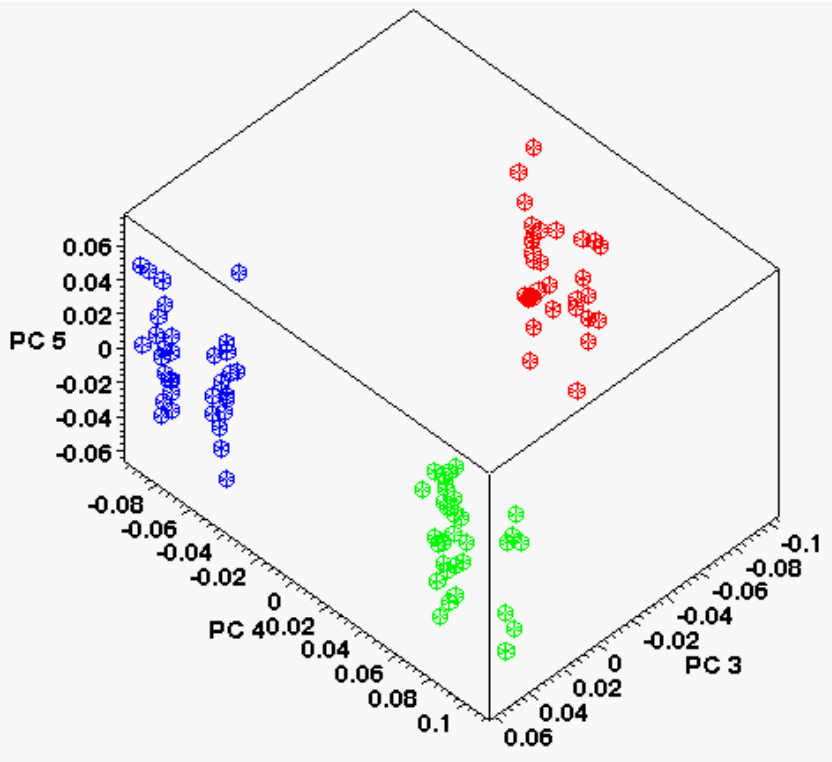
Legend

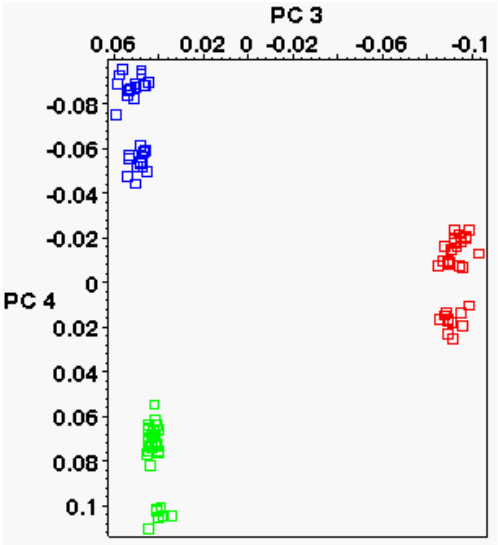
Class 1

Class 2

Class 3

Example 3: Dimensions 3, 4 & 5





Legend

Class 1

Class 2

Class 3

Example 3 Observations

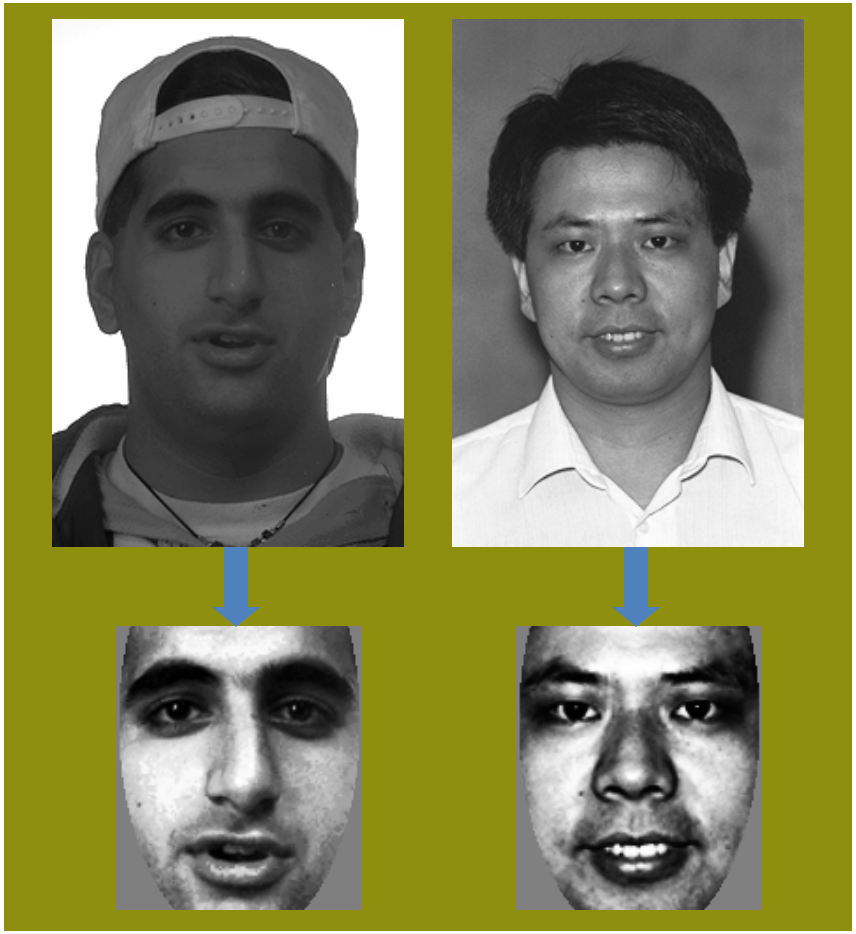
- The first two Principle Components carry no information with respect to image class.
- However, Principle Components 3 and 4 carry all the information necessary for a nearest neighbors classifier

$$[4.775,\ 3.846,\ .4245,\ .3970,\ .07430]$$

1 2 3 4 5

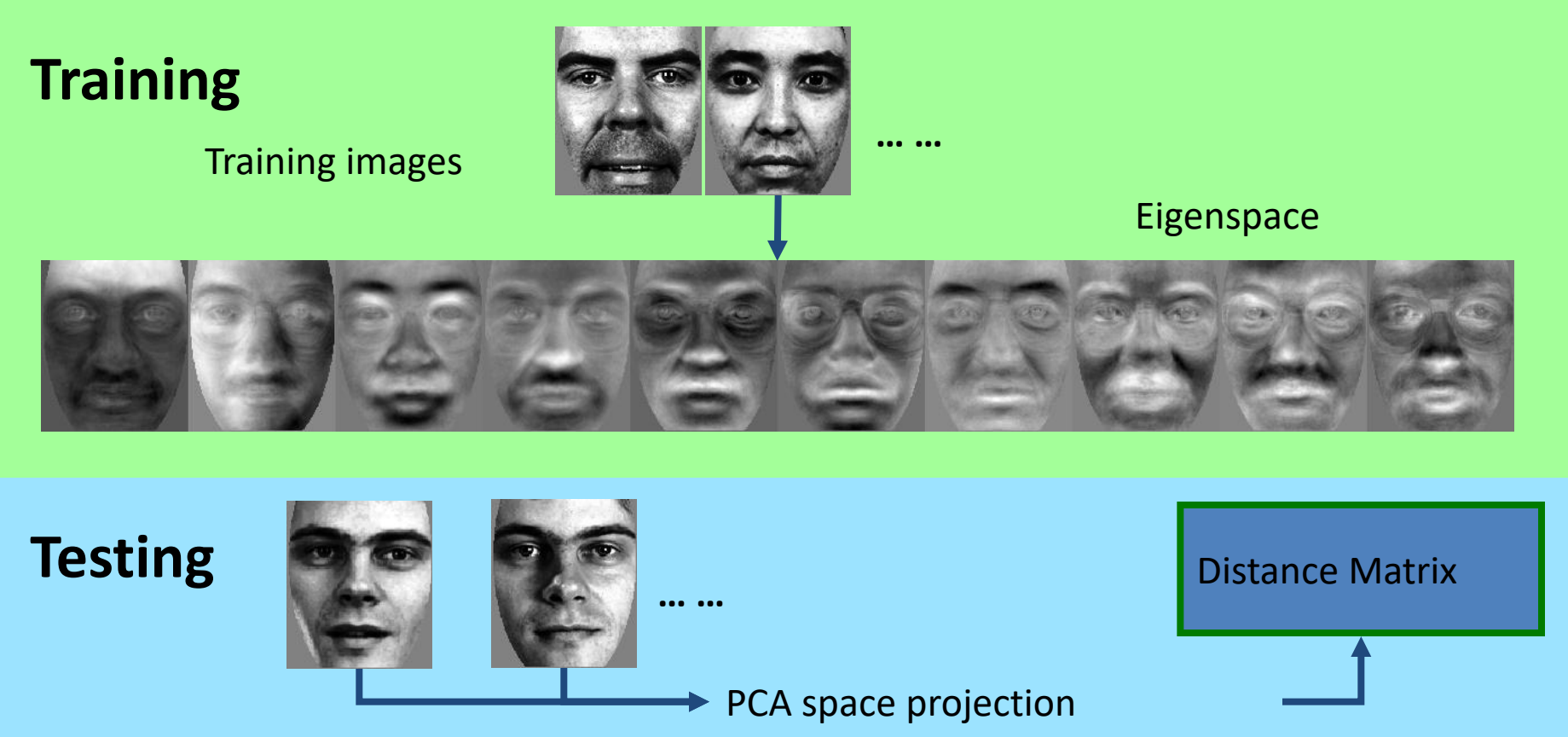
The Eigenvalues, which record variance along each axis, show higher PC's have more variance:

On to Faces - Preprocessing



- Integer to float conversion
  - Converts 256 gray levels to single-floats
- Geometric Normalization
  - Aligns human chosen eye coordinates
- Masking
  - Crop with elliptical mask leaving only face visible.
- Histogram Equalization
  - Histogram equalizes unmasked pixels: 256 levels.
- Pixel normalization
  - Shift and scale pixel values so mean pixel value is zero and standard deviation over all pixels is one.

Standard Eigenfaces



Linear Discriminant Analysis

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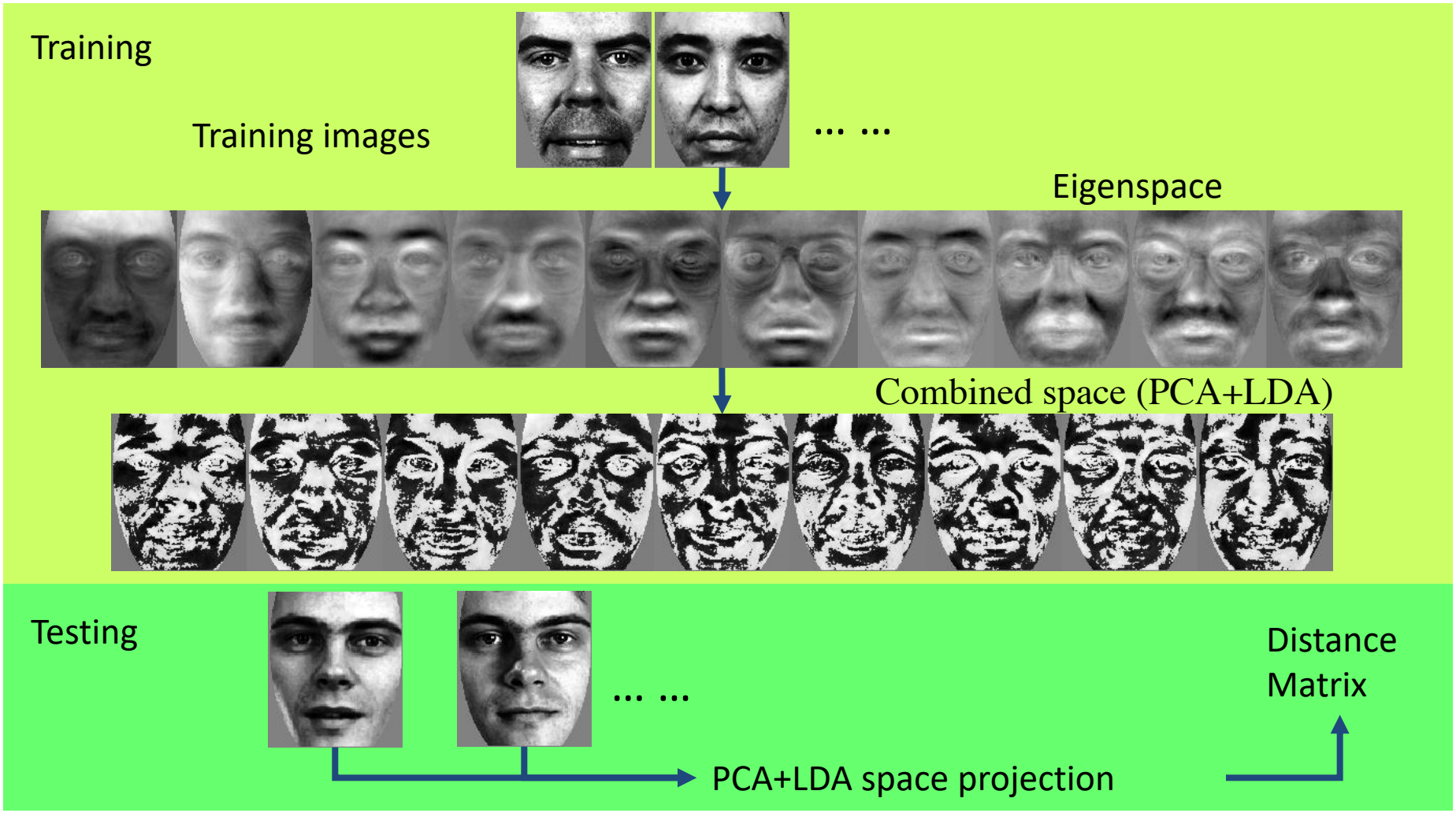
May 30, 2001 - fixing our attention on Gaussian points clouds is that it allows us to develop an intuition for the **geometry** underlying both **PCA** and **LDA** spaces.

The PCA-LDA Variant



More Recently

Local Region PCA Algorithm

- 13 Local Features + Hole Face
- Self Quotient - Lighting Removal
- PCA based whitening

- 250 basis vectors per feature.

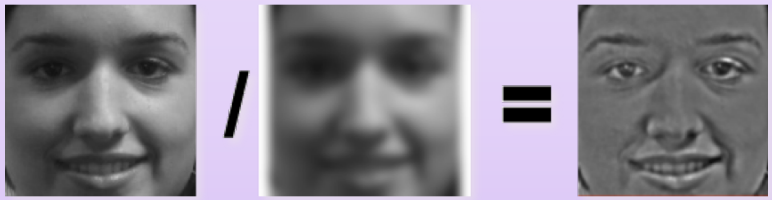
- 3500 total basis vectors.

- Fisher Criterion Weighting

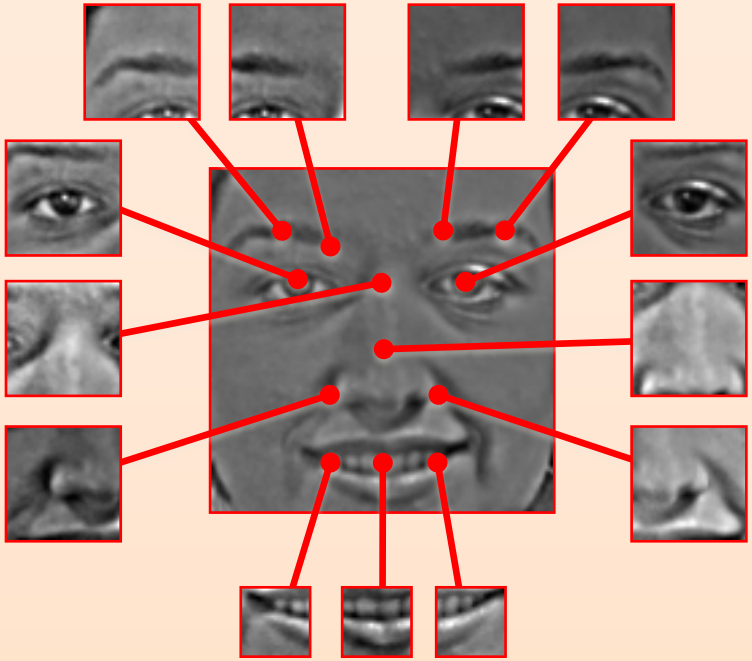
- All features combined

- Similarity based upon Correlation

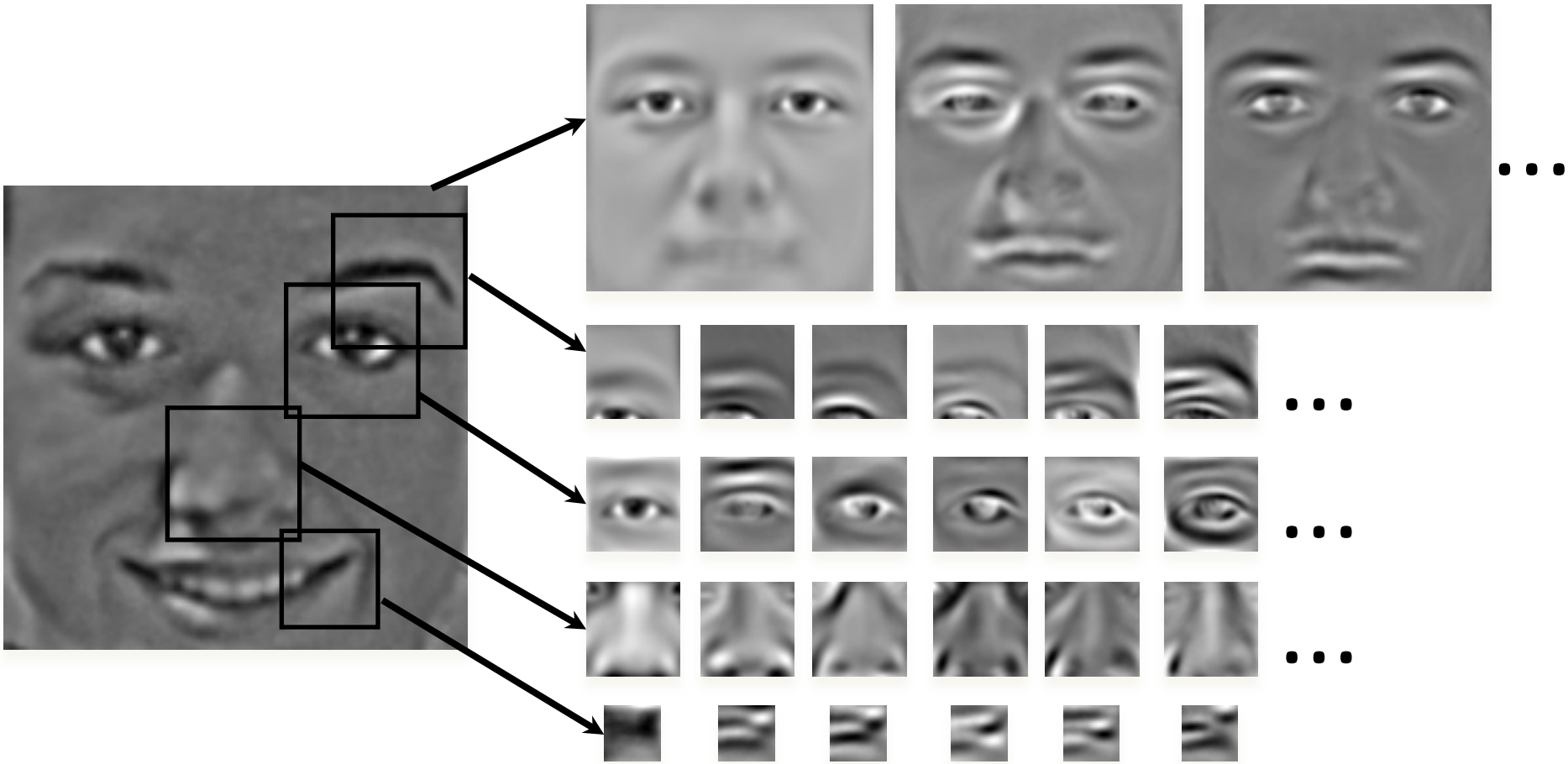
Self Quotient Preprocessing



Local Features



LRPCA generates 3500 Features



Summary

- PCA can be applied to any set of registered images
- It extracts the dimensions of maximum co-variance
  - In some sense, the “structure” of the domain
- Dimensions of co-variance may or may not be related to classification
- No longer competitive by itself, but still a common part of classification strategies in high-dimensional data.