Fault Tolerant Computing

CS 530

Random Testing

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Random Testing

- Random testing, in some form, is common for both hardware or software testing.
- It is sometimes assumed that an “average” fault can represent most faults. In reality some faults are easy to find, while some faults are very hard to find.
- For highly reliable, the real challenge is in finding hard-to-test bugs.
- “Detectability-profile” concept is introduced here.
Random Testing: Outline

• Random Testing (RT): advantages and tradeoffs
• RT vs pseudorandom testing (PR)
• Coverage and detectability profile (DP)
• Hardware and software DPs
• Connection between coverage and faults found?
• High and low testability faults during early & late testing
• Implications of an asymmetric DP
Random Testing

- Extensively used for both hardware and software.
- Ideally each input is selected randomly. PR (Pseudorandom) schemes approximate random.
- Generally quite effective for moderate coverage.
- Disadvantages:
  - Coverage hard to determine a priori.
  - Ineffective for random-pattern-resistant faults.
- Coverage tools: Random (functional) followed by Structural testing.
Random Testing: Advantage

- No test generation using structural information needed.
- Test set-up using comparison:

  ![Diagram](image)

- Alternative: Is response reasonable?

Q: For software testing, how do you know the expected response?
Pseudorandom (PR) Testing

• Unlike true random, reproducible.
• Will not repeat until all combinations applied.
• Generation: usually just-in-time (not stored).
  ▪ Autonomous linear feedback shift register (ALFSR).
  ▪ Cellular automata etc possible.
• Some randomness properties satisfied, but not all.

Ex: Set of vectors with more 1s than 0s is not quite random.
Randomness is hard to achieve

NIST: A Statistical Test Suite for Random and Pseudorandom Number Generators for Cryptographic Applications
Coverage Achieved

- Coverage grows fast in the beginning, saturates near end.
- Is it described by
  - $C(L) = 1 - e^{-aL}$?
  - No, doesn’t fit.
- It is controlled by distribution of detectability of faults.
- Detectability profile (Malaiya & Yang ’84):
  - $H = \{h_1, h_2, \ldots h_N\}$
    - N: total possible vectors
    - $h_k$: number of faults detected by exactly k vectors.
  - Total faults $M = \Sigma h_k$
  - $h_1$: number of least testable faults

Ex: Circuit with higher $h_1$ would be harder to test.
Ex: Find Detectability Profile

Example:

<table>
<thead>
<tr>
<th></th>
<th>a-0</th>
<th>a-1</th>
<th>b-0</th>
<th>b-1</th>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Minimum set of faults

Answer: \( h_k \): number of faults detected by exactly \( k \) vectors. Thus \( h_1 = 5 \) faults, \( h_3 = 1 \) fault. Hence \( H = (h_1, h_3) = \{5, 1\} \)

Question: What is the probability that a random test will test for \( c \) s-a-1?
Detection Probability

• **Detection probability:** if there are \( N \) distinct possible vectors, and if a fault is detected by \( k \) of them, then its detection probability is \( \frac{k}{N} \)

• A fault with detection probability \( \frac{1}{N} \) would be hardest to test, since it is tested by only one specific test and none other.
Detectability Profiles: Ex

- **CECL Full adder**
  Inputs=4 (N=16), M=90
  H=(h₁,h₂,h₃,h₄,h₅,h₆,h₈) = (1,11,2,43,21,4,8)

- **Schneider’s counterexample circuit:**
  Inputs= 4 (N=16), M=44
  H=(h₁,h₂,h₃,h₁₄)=(23,19,1,1)

*Schneider’s counterexample circuit* has 23 hard to test faults. A random vector has probability 1/16 to detect any one of them.
Coverage with L random vectors

- $h_k$ out of $M$ defects detectable by exactly $k$ vectors: detection probability $k/N$
- $P\{\text{a defect with } dp \frac{k}{N} \text{ not detected by a vector}\} = \left(1 - \frac{k}{N}\right)$
- $P\{\text{a defect with } dp \frac{k}{N} \text{ not detected by } L \text{ vectors}\} = \left(1 - \frac{k}{N}\right)^L$
- Of $h_k$ faults, expected number not covered is $\left(1 - \frac{k}{N}\right)^L h_k$
- Expected test coverage with $L$ vectors

$$C(L) = 1 - \sum_{k=1}^{N} \left(1 - \frac{k}{N}\right)^L \frac{h_k}{M}$$
Coverage Obtained by L Vectors

- For PR tests (McClusky 87)
  
  \[ C(L) = 1 - \sum_{k=1}^{N-L} \frac{C_k}{N} \frac{h_k}{M} \]
  
  \[ \approx 1 - \sum_{k=1}^{N} \left(1 - \frac{k}{N}\right)^L \frac{h_k}{M} \] (for Random)

- For large L, terms with only low k (i.e. faults that are hard to test) have an impact. Thus only lower elements of H need to be estimated.

- For CECL Full Adder,
  
  \[ C(15) = 1 - [4.2 + 16.4 + 0.9 + 6.3 + 0.84 + 0.03 + 0 + ...] \cdot 10^{-3} \]

Pseudorandom (PR): a vector cannot repeat, unlike in true Random.

More in Appendix 1
Detectability Profile: software

- Regardless of initial profile, after some initial testing, the profile will become asymmetric
- In the early development phases, inspection and early testing are likely to remove most easy to test bugs, while leaving almost all hardest to test bugs still in.
Detectability Profile: software

- Adam’s Data for a large IBM software product

Notice: Fewer bugs with higher detection rates

Detectability Profile: Software

- Software detectability profile is exponential
- Justification: Early testing will find & remove easy-to-test faults.
- Testing methods need to focus on hard-to-find faults.

As testing time progresses, more of the faults are clustered to the left.

![Graph showing detectability profile with hard to test and low hanging fruit markers.]

Hard to test  Low hanging fruit
Implications of Asymmetrical DP

- Faults are not alike, and an “average” fault does not represent a hard-to-test fault.
- A fault injected artificially typically does not represent a hard-to-test fault.
- Faults found early during testing are not a good sample of faults that will be found later during testing.
Implications of Asymmetrical DP

- Fault seeding
- Fault sampling
- Fault exposure ratio
Implications: Fault Seeding

- A program has $x$ defects. We want to estimate $x$.
- Seed $j$ new faults.
- Do some testing. Let faults found be $j_1$ seeded faults and $x_1$ original faults.
- Assuming $j_1/j = x_1/x$ we get $x = x_1 \frac{j}{j_1}$
- However, in reality the $x$ faults include harder faults to test,
  $$\frac{j_1}{j} > \frac{x_1}{x} \quad hence \quad x > \frac{x_1 j}{j_1}$$
Implications: Estimation by Inspection Sampling

- Software with $x$ bugs is inspected by two separate teams that find $x_1$ and $x_2$ bugs respectively, of which $x_3$ are shared.

- Assuming $x_1/x = x_3/x_2$ we get
  \[ x = \frac{x_1 x_2}{x_3} \]

- However actually since $x$ includes more harder to test faults,
  \[ \frac{x_3}{x_2} > \frac{x_1}{x} \text{ hence } x > \frac{x_1 x_2}{x_3} \]
Implications: fault exposure ratio

Let \( N(t) \) be the number of bugs at time \( t \) during testing, then if \( a \) is a parameter,

\[
\frac{dN(t)}{dt} = -aN(t)
\]

If \( a \) is constant, then \( N(t) = N(0)e^{-at} \) [expo SRGM]

However in random testing \( a \) should decline as faults get harder to find.

If testing is intelligent, then \( a \) can rise, which can give rise to Logarithmic SRGM.

Don’t worry about this here, we will come to it when we will study software reliability.
References

Appendix

Ex: Coverage for CECL adder

- 16 vectors (0000 to 1111), 90 potential defects (transistor level)

- A fault with det prob 1/16 has probability 0.0625 to be detected with 1 test, while fault with det prob 8/16 has 0.5 (i.e. 50%) of getting tested by it.

- With 20 vectors, 28% of faults with det prob 1/16 will still be left undetected, while those with det prob 8/16 will almost certainly be found.

- Table next gives the coverage obtained for faults with specific detectability values.
**Ex: C(L) and components for CECL Full Adder**

**CECL full adder**

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<th>Hk</th>
<th>1</th>
<th>11</th>
<th>2</th>
<th>43</th>
<th>21</th>
<th>4</th>
<th>8</th>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
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<table>
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</tr>
<tr>
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</tr>
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<td>10</td>
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<td>0.6202</td>
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<tr>
<td>20</td>
<td>0.7249</td>
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</table>

After 20 vectors:

| covered | 0.72 | 10.24 | 1.97 | 42.86 | 20.99 | 4.00 | 8.00 |
| remaining | 0.28 | 0.76 | 0.03 | 0.14 | 0.01 | 0.00 | 0.00 |
Coverage of faults with different detectability values

The plot in the next slide for CECL Full Adder shows that

• Faults with high detection probability get covered soon,
• while those with low detection probability are resistant to random testing.
Coverage of partitions

![Graph showing coverage of partitions with test length L on the x-axis and partition coverage on the y-axis. The graph includes two lines: one for k=1 and another for k=8.]
Shift in profile with progress in testing

Next slide for CECL Full Adder
• Assume that a fault is removed from consideration when found
• X-axis is k (k=1 hardest to find)
• Plot shows that at the beginning there are nearly 50% of the faults with det prob k/N = 4/16.
• After 20 vectors, more than 60% of the remaining faults have det prob 2/16.
• “Low hanging fruit” get picked quicker.
Shift in profile with progress in testing