



Fault Tolerant Computing

CS 530

Software Reliability Growth

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Software Reliability Growth: Outline

- Testing approaches
- Operational Profile
- Software Reliability Growth Models
 - Exponential
 - Logarithmic
- Model evaluation: error, bias
- Model usage
 - Static estimation before testing
 - Making projections using test data

Software Reliability Growth Models

- This field is the classical part of “**Software Reliability Engineering**” (SRE).
- During testing and debugging, the number of remaining bugs reduces, and the bug finding rate tends to drop.
- **When to Stop Testing Problem**: Given a history of bug finding rate, when will it drop below an acceptable limit, so that the software can be released.

Test methodologies

- **Static** (review, inspection) vs. **dynamic** (execution)
- **Test views**
 - **Black-box** (functional): input/output description
 - **White box** (structural): implementation used
 - **Combination**: *white after black*
- **Test generation**
 - **Partitioning** the input domain
 - **Random/Antirandom/Deterministic**
- **Usual assumption: the test method does not change during testing.**
 - In practice testing approach does change, which causes some statistical fluctuations.

Input mix: Test Profile

- The inputs to a system can represent different types of operations. The input mix called “**Profile**” can impact effectiveness of testing.
- For example a Search program can be tested for text data, numerical data, data already sorted etc. If most testing is done using numerical data, more bugs related to text data may remain unfound.

Input Mix: Testing “Profile”

- **The ideal Profile (input mix) will depend on the objective**
 - A. Find bugs fast? or
 - B. Estimate operational failure intensity?
- A. Best mix for efficient bug finding ([Li & Malaiya](#)’ 94)
 - Quick & limited testing: Use *operational profile* (next slide)
 - High reliability: *Probe input space evenly*
 - Operational profile will not execute rare and special cases, the main cause of failures in highly reliable systems.
 - In general: Use combination
- B. For *acceptance testing*: Need Operational profile

N. Li and Y.K. Malaiya, On Input Profile Selection for Software Testing, Proc. Int. Symp. Software Reliability Engineering, Nov. 1994, pp. 196-205.

H. Hecht, P. Crane, Rare conditions and their effect on software failures, Proc. Annual Reliability and Maintainability Symposium, 1994, pp. 334-337

Operational Profile

- **Profile:** set of disjoint actions, operations that a program may perform, and their probabilities of occurrence.
- **Operational profile:** probabilities that occur in actual operation
 - Begin-to-end operations & their probabilities
 - Markov: states & transition probabilities
- There may be multiple operational profiles.
- Accurate operational profile determination may not be needed.

Operational Profile Example

- Assume PhoneFollower software that handles incoming calls to a PABX unit.
- Incoming call types & other operations (total 7 types) are monitored to estimate get their probabilities (next slide).
- 74% of the calls were *voice calls*. In order to achieve better resolution, they were further divided into 5 type (next slide).
- The resulting Operational profile would have $5+6 = 11$ types of operations, with probabilities ranging from 0.18 (18%) to 0.000001.

Note that the code needed for Failure recovery is only rarely executed.

Operational Profile Example

- *“Phone follower” call types (Musa)*

A	Voice call	0.74
B	FAX call	0.15
C	New number entry	0.10
D	Data base audit	0.009
E	Add subscriber	0.0005
F	Delete subscriber	0.000499
G	Failure recovery	0.000001

A1	Voice call, no pager, answer	0.18
A2	Voice call, no pager, no answer	0.17
A3	Voice call, pager, voice answer	0.17
A4	Voice call, pager, answer on page	0.12
A5	Voice call, pager, no answer on page	0.10

Modeling Reliability Growth

- Testing cost can be 60% or more
- Careful planning to release by target date
- Decision making using a *software reliability growth model (SRGM)*. Obtained using
 - Analytically using assumptions, or and
 - Based on experimental observation
- A **model** describes a **real process** approximately
- Ideally should have good predictive capability and a reasonable interpretation

Exponential Reliability Growth Model

- Most common and easiest to explain model. From 1970s
- Notation:
 - **Total expected faults** detected by time t : $\mu(t)$
 - **Failure intensity: fault detection rate** $\lambda(t)$
 - **Undetected defects present at time t :** $N(t)$
- **By definition, $\lambda(t)$ is derivative of $\mu(t)$. Hence**

$$\begin{aligned}\lambda(t) &= \frac{d}{dt} \mu(t) \\ &= -\frac{d}{dt} N(t)\end{aligned}$$

Since faults found are no longer undetected

Exponential SRGM Derivation Pt 1

■ Notation

- T_s : average single execution time
- k_s : expected fraction of faults found during T_s
- T_L : time to execute each program instruction once

$$-\frac{dN(t)}{dt} T_s = k_s N(t)$$

Key
assumption

$$-\frac{dN(t)}{dt} = \frac{K}{T_L} N(t) = \beta_1 N(t)$$

Notation: Here we replace K_s and T_s by more convenient K and T_L .

where $K = k_s \frac{T_L}{T_s}$ is **fault exposure ratio**

Exponential SRGM Derivation Pt 2

- We get

$$N(t) = N(0) e^{-\beta_1 t}$$

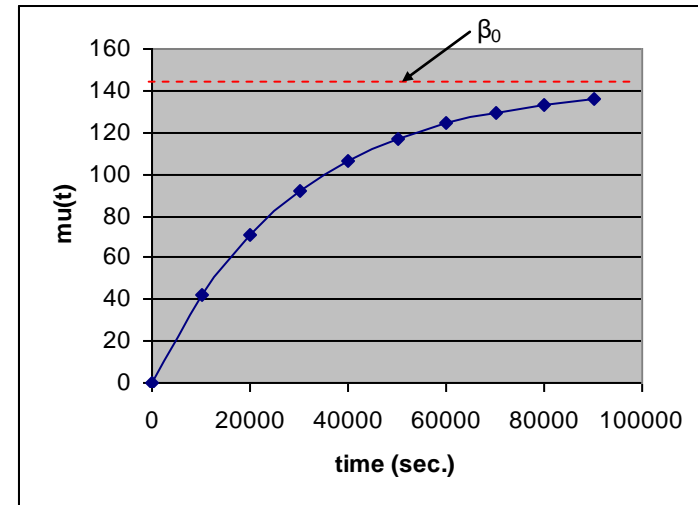
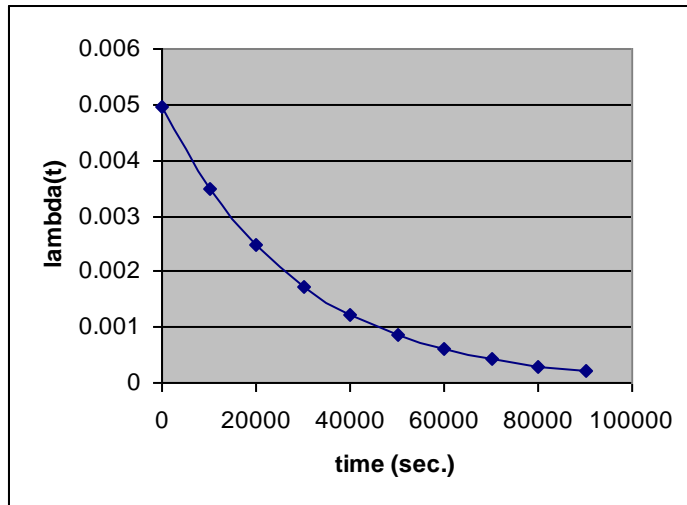
$$\mu(t) = \beta_0 (1 - e^{-\beta_1 t})$$

$$\lambda(t) = \beta_0 \beta_1 e^{-\beta_1 t}$$

The 2 equations contain the same information.

- For $t \rightarrow \infty$, total $\beta_0 = N(0)$ faults would be eventually detected. A “*finite-faults-model*”.
- Assumes no new defects are generated during debugging.
- Proposed by Jelinski-Muranda ‘71, Shooman ‘71, Goel-Okumoto ‘79 and Musa ‘75-’ 80. also called Basic.

Exponential SRGM



The plots show $\lambda(t)$ and $\mu(t)$ for $\beta_0=142$ and $\beta_1=3.5 \times 10^{-5}$. Note that $\mu(t)$ asymptotically approaches 142.

A Basic SRGM (cont.)

- Note that **parameter** β_1 is given by:

$$\beta_1 = \frac{K}{T_L} = \frac{K}{(S \cdot Q \cdot \frac{1}{r})}$$

- S: source instructions,
- Q: number of object instructions per source instruction typically between 2.5 to 6 (see page 7-13 of [Software reliability Handbook, sec 7](#))
- r: object instruction execution rate of the computer
- K: *fault-exposure ratio*, range 1×10^{-7} to 10×10^{-7} , (t is in CPU seconds). Assumed constant here*.
- Q, r and K should be relatively easy to estimate.

*Y. K. Malaiya, A. von Mayrhauser and P. K. Srimani, "An examination of fault exposure ratio," in IEEE Transactions on Software Engineering, vol. 19, no. 11, pp. 1087-1094, Nov 1993

SRGM : “Logarithmic Poisson”

- Many SRGMs have been proposed.
- **Another model Logarithmic Poisson** model, by **Musa-Okumoto**, has been found to have a good predictive capability

$$\mu(t) = \beta_0 \ln(1 + \beta_1 t) \qquad \lambda(t) = \frac{\beta_0 \beta_1}{1 + \beta_1 t}$$

- Applicable as long as $\mu(t) \leq N(0)$. Practically always satisfied. Term *infinite-faults-model* misleading.
- Parameters β_0 and β_1 don't have a simple interpretation. An interpretation has been given by Malaiya and Denton ([What Do the Software Reliability Growth Model Parameters Represent?](#)).

Comparing Models


- Goodness of fit: may be misleading
- Predictive capability:
 - Data points: (λ_i, t_i) , $i = 1$ to n
 - Total defects found: D , estimated at i : D_i

$$\text{Average error : AE} = \frac{1}{n} \sum_{i=1}^{n-1} \left| \frac{D_i - D}{D} \right|$$

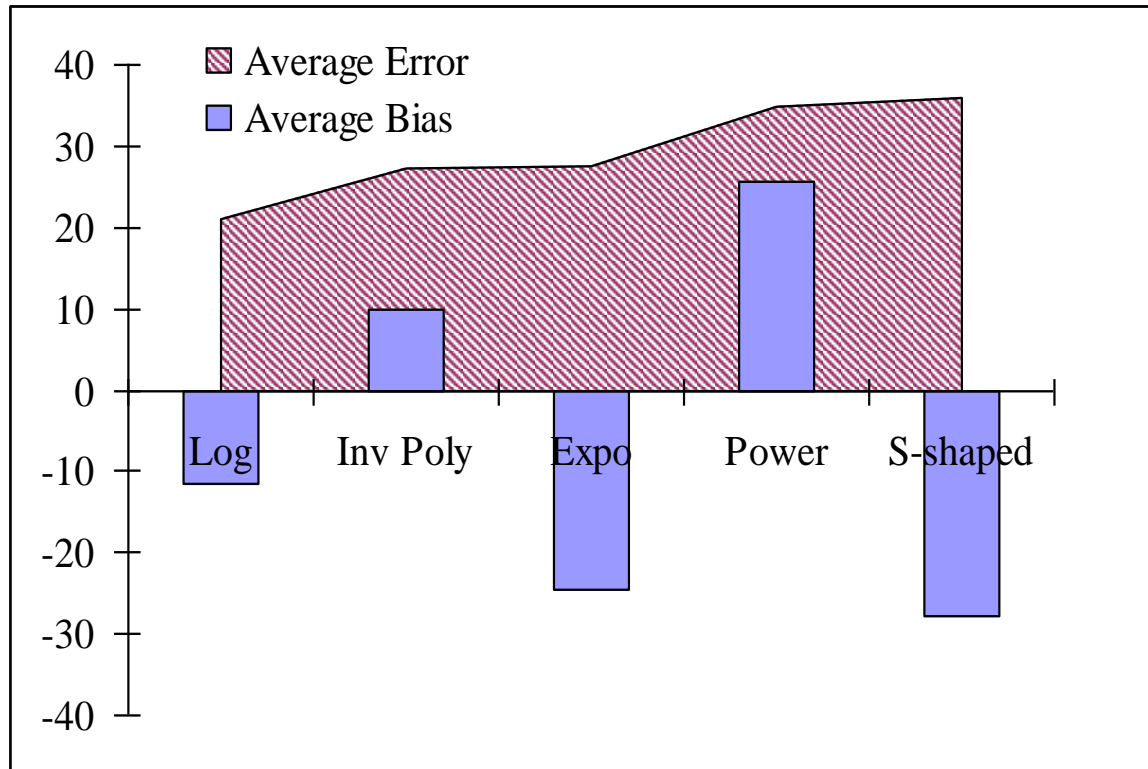
$$\text{Average bias : AB} = \frac{1}{n} \sum_{i=1}^{n-1} \frac{D_i - D}{D}$$

- We used many datasets from diverse projects for comparing different models.

Comparing models

- Next slide shows the result of a comparison using test data from a number of diverse sources. 
- The Logarithmic Poisson model is most accurate.
- The Exponential model is moderately accurate.
- Both the Logarithmic Poisson and the Exponential models tend to underestimate the number of defects that will eventually be found.
- Inverse Polynomial, Power and S-shaped models are not discussed here, you can find them in the literature.

Bias in SRGMs



- [Malaiya, Karunanithi, Verma \('90\)](#)

Using an SRGM

- An SRGM can be used in two ways
 - For preliminary planning, even before testing begins (provided you can estimate the parameters)
 - During testing: You can fit the available test data to make projections.
- We'll see examples of both next.

SRGM: Use for Preliminary Planning

- Example:
 - initial defect density estimated 25 defects/KLOC
 - 10,000 lines of C code
 - computer 70 million object instructions per second
 - *fault exposure ratio* K estimated to be 4×10^{-7}
 - **Task:** Estimate the testing time needed for defect density 2.5/KLOC
- Procedure:
 - Find β_0, β_1
 - Find testing time t_1

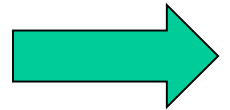


SRGM: Preliminary Planning (cont.)

- From exponential model

$$\beta_o = N(0) = 25 \times 10 = 250 \text{ defects,}$$

$$\begin{aligned}\beta_1 &= \frac{K}{(S.Q. \frac{1}{r})} = \frac{4.0 \times 10^{-7}}{10,000 \times 2.5 \times \frac{1}{70 \times 10^6}} \\ &= 11.2 \times 10^{-4} \text{ per sec}\end{aligned}$$



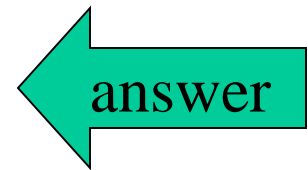
SRGM: Preliminary Planning (cont.)

- Reliability at release depends on

$$\frac{N(t_1)}{N(0)} = \frac{2.5 \times 10}{25 \times 10} = \exp(-11.2 \times 10^{-4} \cdot t_1)$$

Note $N(t_1)$ is the total number of defects at the end of testing, which is *defect density* \times *size*
 $= 2.5/\text{KLOC} \times 10 \text{ KLOC}$

$$t_1 = \frac{-\ln(0.1)}{11.2 \times 10^{-4}} = 2056 \text{ sec. (CPU time)}$$




$$\begin{aligned} \lambda(t_1) &= 250 \times 11.2 \times 10^{-4} e^{-11.2 \times 10^{-4} t_1} \\ &= 0.028 \text{ failures/ sec} \end{aligned}$$

SRGM: Preliminary Planning (cont.)

- For the same environment, product $\beta_1 \times S$ is constant, since β_1 is inversely proportional to S . For example,
 - If for a prior 5 KLOC project β_1 was 2×10^{-3} per sec.
 - Then for a new 15 KLOC project, β_1 can be estimated as $2 \times 10^{-3}/3 = 0.66 \times 10^{-3}$ per sec.
- Value of fault exposure ratio (K) may depend on initial defect density and testing strategy (Li, Malaiya '93).

SRGM: During Testing

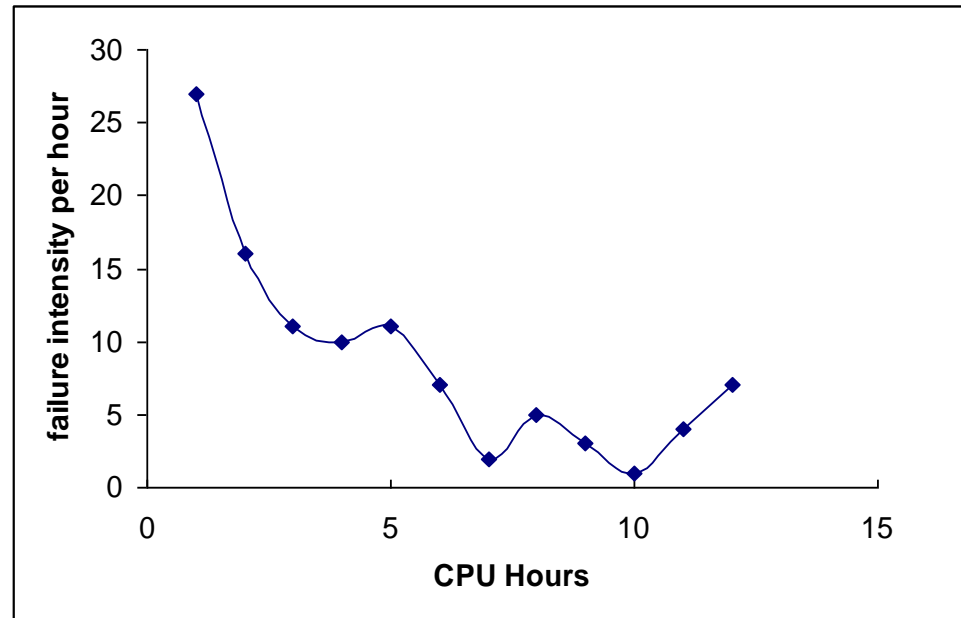
- Collect and pre-process data:
 - To extract the long-term trend, data needs to be smoothed
 - *Grouped* data: test duration intervals, average failure intensity in each interval.
- Select a model and determine parameters :
 - past experience with projects using the same process
 - exponential and logarithmic models often good choices
 - model that fits early data well, may not have the best predictive capability
 - parameters estimated using *least square* or *maximum likelihood*
 - parameter values used when *stable* and *reasonable* 

SRGM: During Testing (cont.)

- Compute how much more testing is needed:
 - fitted model to project additional testing needed
 - desired failure intensity
 - estimated defect density
 - recalibrating a model can improve projection accuracy
 - Interval estimates can be obtained using statistical methods.

Example: SRGM with Test Data

CPU Hours	Failures
1	27
2	16
3	11
4	10
5	11
6	7
7	2
8	5
9	3
10	1
11	4
12	7



- Target failure intensity 1/hour (2.78×10^{-4} per sec.)

Example: SRGM with Test Data (cont.)

- Fitting we get

$$\beta_0 = 101.47 \text{ and } \beta_1 = 5.22 \times 10^{-5}$$

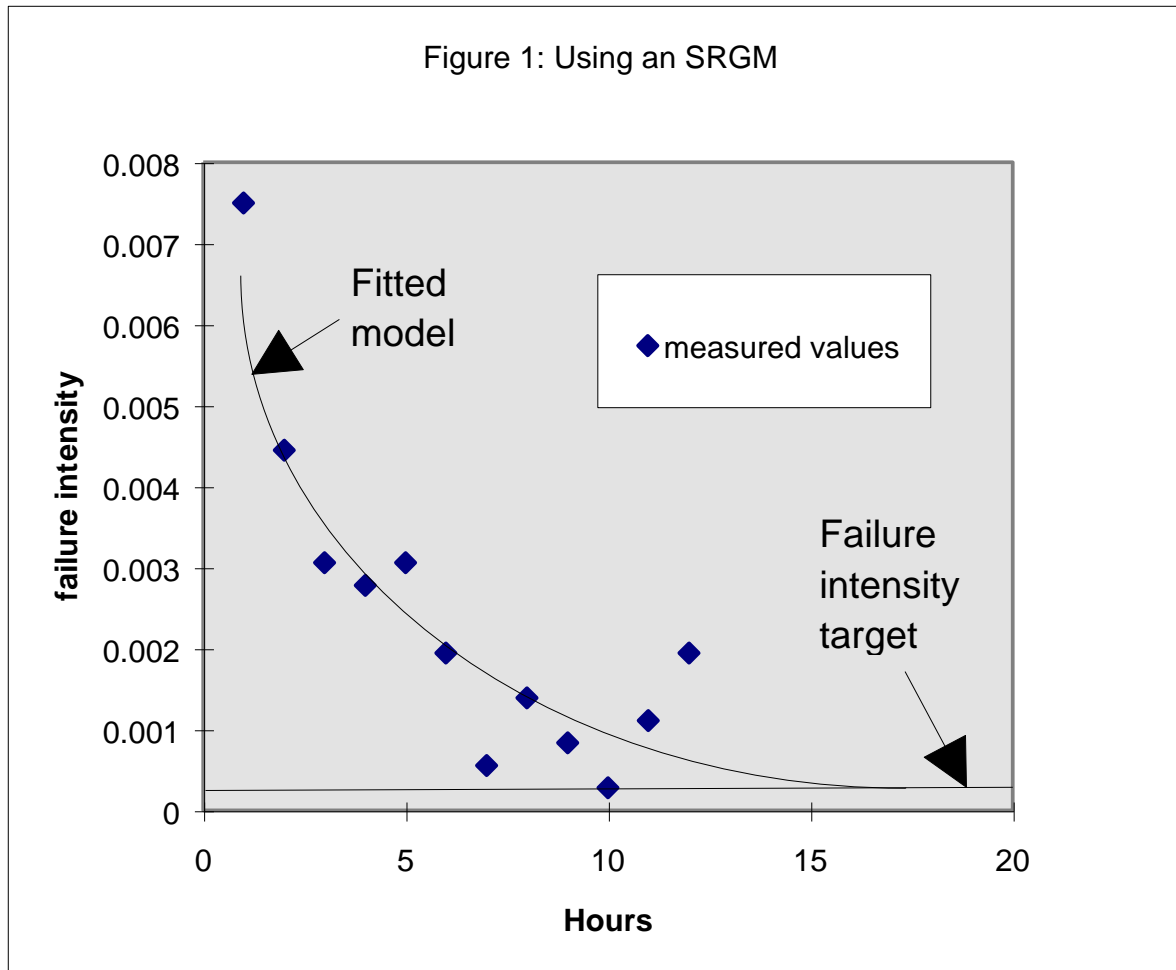
- stopping time t_f is then given by:

$$2.78 \times 10^{-4} = 101.47 \times 5.22 \times 10^{-5} e^{-5.22 \times 10^{-5} \times t_f}$$

- yielding $t_f = 56,473$ sec., i.e. 15.69 hours

Note: The exact values of the parameter values estimated depend on the numerical methods used.

Example: SRGM with Test Data (cont.)



Example: SRGM with Test Data (cont.)

- Accuracy of projection:
 - Experience with Exponential model suggests
 - estimated β_0 tends to be lower than the final value
 - estimated β_1 tends to be higher
 - true value of t_f should be higher. Hence 15.69 hours should be used as a lower estimate.
- Problems :
 - test strategy changed: spike in failure intensity
 - smoothing
 - software under test evolving - continuing additions
 - Drop or adjust early data points

Test Compression factor

- β_1 measures test effectiveness,
 - higher during testing for debugging,
 - lower during operation.
 - Test compression factor = β_{1t} / β_{1op}
 - Similar to accelerated testing for hardware
- Needs to be estimated empirically
 - Musa's estimates: 8-20 for systems with 10^3 to 10^9 states.
 - Needs further research
- CPU time vs calendar time

For further reading

- [Software Reliability Assurance Handbook](#) by Lakey and Neufelder
- Y.K. Malaiya, "[Software Reliability Management](#)," Encyclopedia of Library and Information Sciences, Taylor and Francis, Editor: S. Lee, Third Edition, 1: 1, 4901 — 4912, February 2010.