
Fault Tolerant Computing
CS 530
Reliability Analysis Pt 2
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Reliability Analysis: Continuation

- Note that a parallel configuration is an ideal redundant system. It assumes that a correctly operating unit can always be identified.
- Here we examine Triple-Modular Redundancy, a realistic scheme that has been used for both hardware and software. It is a special case of the combinatorial k-out-of-n system.
- We also examine use of switching to switch out a bad unit.

k-out-of-n Systems

- Assumption: we have n identical modules with statistically independent failures.
- k-out-of-n system is operational if
  k of the n modules are good.
- System reliability then is

$$R_{k/n} = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

Note that you can choose i good system out of n in {n choose i} (i.e. nCi) ways.

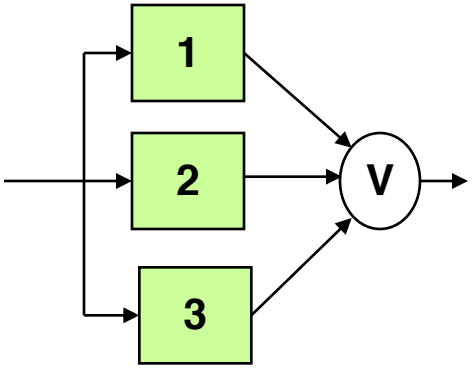
Also recall that nCi = n!/[i!(n-i)!]

nCi is also written like the equation on the left.

Where p is the probability that one unit is good. $R_{k/n}$ is the summations of the probabilities of all good combinations.

Triple Modular Redundancy

- **Popular high-reliability scheme: 2-out-of-3 system**
- **Output is obtained using a majority voter**



$$R_{TMR} = \sum_{i=2}^{3} \binom{3}{i} R^i (1-R)^{3-i}$$

$$= 3R^2(1-R) + R^3$$

$$= 3R^2 - 2R^3$$

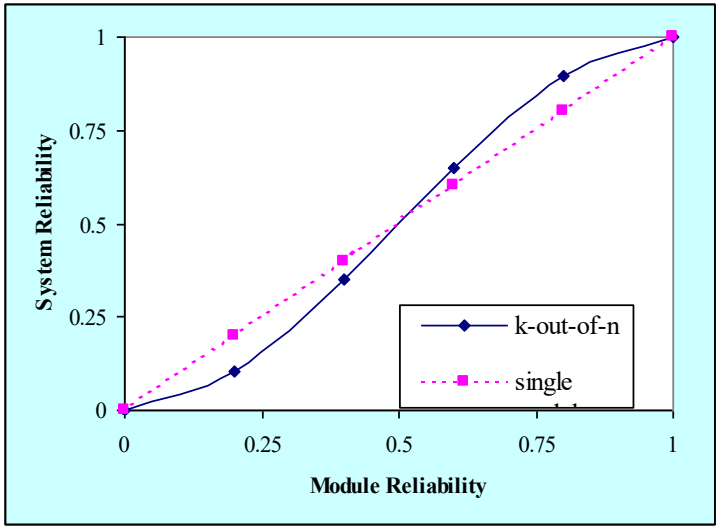
Where R is the reliability of a single module. This assumes that the voter is perfect, a reasonable assumption if the voter complexity is much less than an individual module.

Triple Modular Redundancy

Here is a plot of the system reliability, when the individual module reliability varies between 0 to 1.

Note that if the reliability of an individual module is less than 0.5, it is more likely to be bad. Having several such modules, and taking majority vote, will actually make the system less reliable, as you see in the figure.

A political application of the principle: majority-based democracy works only if the individuals are more likely to make the right decision than wrong!



Story of Tagore Family

From Barnes and Noble Coffee Shop

TMR: Permanent Failures

$$\text{Let } R = e^{-\lambda t}$$

$$R_{TMR}(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$MTTF = \int_0^\infty R_{TMR}(t)dt$$

$$= \int_0^\infty (3e^{-2\lambda t} - 2e^{-3\lambda t})dt$$

$$= \frac{5}{6\lambda} \quad (\text{single module MTTF} : \frac{1}{\lambda})$$

- When is a single module as reliable as TMR?

  Solving $3R^2 - 2R^3 = R$

  we get $R_{cross} = 0.5$.

  TMR worse after $R < 0.5$!

Thus TMR has a lower MTTF than a single module!

MTTF may not be a good measure when very high reliability levels are maintained.

TMR

- Mission time

$$R_{Th} = 3e^{-2\lambda t_m} - 2e^{-3\lambda t_m}$$

A numerical solution for t_m can be obtained iteratively.

- $Ex: \lambda = 1/\text{year}, R_{Th} = 0.95$

	$MTTF$	t_m
single	$1yr$	0.05
TMR	0.83	0.145

Thus TMR mission time is much better.

- Temporary faults: steady state

$$A_{TMR} = 3A^2 - 2A^3, A = \frac{\mu}{\lambda + \mu}$$

- $Ex: \frac{\lambda}{\mu} = 0.01 \Rightarrow A = 0.9901$

$$\Rightarrow \overline{A} = 0.01$$

$$A_{TMR} = 0.9997 \Rightarrow \overline{A}_{TMR} = 0.0003$$

Thus TMR can greatly reduce down-time in presence of temporary faults.

TMR: implementation issues

- **Hardware: 3 identical processors running either**
  - **Synchronized with each other**
  - **Not synchronized. Voter has to wait until comparable results from all 3 are available.**
- **Software: 3 independently implementations of the same specifications**
  - **The hope is that the failures are statistically independent. In practice, there is some correlation. We will examine the impact later.**

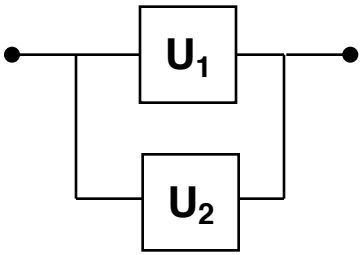
Switching and "Coverage"

- **If a module is suspected of being bad, it can be switched out, and the process is moved to a good module.**
- **Switching successfully requires**
  - **Identifying that the module is bad**
    - **By have some self-test capability in it**
      - Or by comparing its output with that of an identical module
  - **Successfully restarting the process on another module**
  - **"Coverage" is the probability that these two will be done successfully.** (**This coverage has nothing to do with test coverage. I wish they had used another term for this probability).**

Primary and Backup Units

- This popular scheme is an example of a parallel configuration.
  - There is a primary unit that participates in the process.
  - The backup (spare) is used if the primary has failed.
    - When primary fails, the uncorrupted process needs to be initiated on the backup.
    - Sometimes the backup runs a redundant shadow process, so that it is ready when the primary fails.
  - This is sometimes called "dynamic redundancy". It is used when a brief interruption is acceptable.
- Next, we analyze such a system. U_1 is primary and U_2 is secondary, with reliabilities R1 and R2.

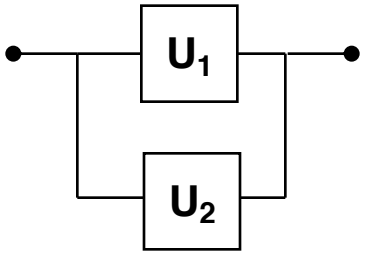
Reliability of Primary/Backup system with imperfect "Coverage"



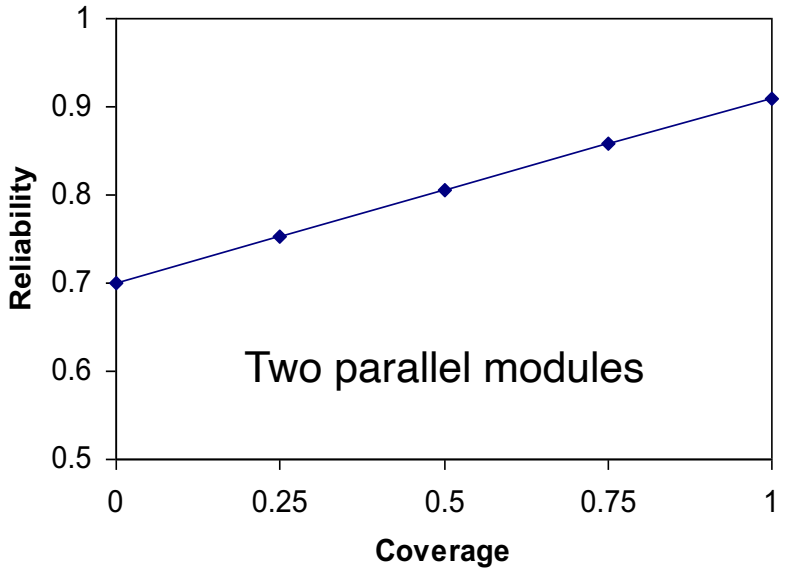
$$R_s = P\{U_1\ good\} +$$
$$P\{U_2\ has\ taken\ over\ |\ U_1\ failed\}P\{U_1\ failed\}$$
$$= R_1 + R_2C(1-R_1)$$
$$where\ \mathrm{C} = \mathrm{P}\{\text{failure detected and successful switchover}\}$$

- Failure detection: requires concurrent detection. This needs some kind of redundancy.
- Switchover requires:
  - good state loaded in U_2.
  - Process restarted.

Imperfect Coverage: Example



- $R_s = R_1 + R_2C(1 - R_1)$
- Assuming $R_1 = R_2 = 0.7$
- In general $R_s = R_m \sum_{i=0}^{n-1} C^i (1 - R_m)^i$



Note that better coverage improves the reliability. When coverage =1, full potential of the parallel configuration is achieved.

Using TMR + Spares

- A TMR will mask a single error, it will be "contained" by the voter. Thus it can be called a "static redundancy". Used when no interruptions are desired.
- You can combine static and dynamic redundancy ("hybrid") by having some spares in addition to the TMR core with three voting modules.
- A permanent failure in the core can be fixed by switching in a spare.
- The system will work as long as we have two good modules in the TMR core.

TMR+Spares

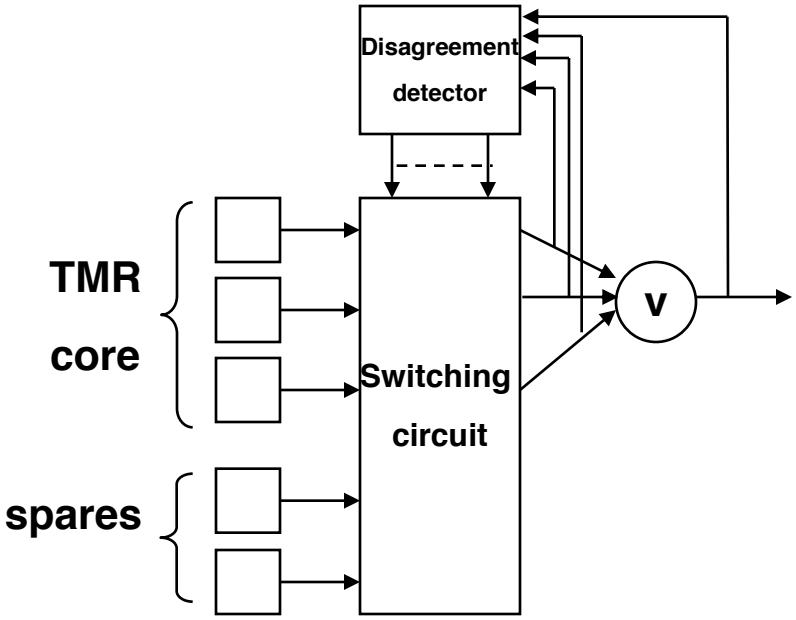
- TMR core, n-3 spares (assume same failure rate)
- Scheme A: System failure when all but one modules have failed. If we start with 3 in the core and 2 spares, the sequence will be

  $3+2 \rightarrow 3+1 \rightarrow 3+0 \rightarrow 2+0 \rightarrow$ failure
- Reliability of the system then is

  $R_s=R_{sw}[1-nR(1-R)^{n-1}-(1-R)^n]$

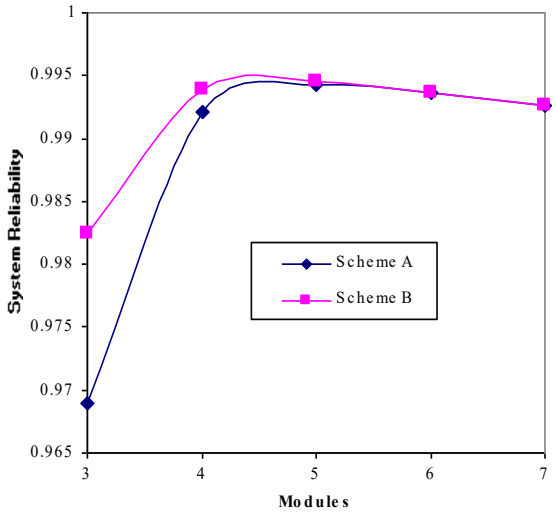
  Where R is reliability of a single module and R_{sw} is the reliability of the switching circuit overhead.
- R_{sw} should depend on total number of modules n, and relative complexity of the switching logic.
- Let us assume that $R_{sw}=(R^a)^n$,

  where a is measure of relative complexity, generally a <<1. Then
- $R_s=R^{an}[1-nR(1-R)^{n-1}-(1-R)^n]$



Example: TMR+Spares

- Ex: R=0.9, $a=10^{-2}$. How many modules will give us the best reliability?
- $R_s=R^{an}\,[1-nR(1-R)^{n-1}-(1-R)^n]$
- The plot of scheme A show that the optimal $n_{max}\approx 4$, i.e. one spare in addition of a TMR core.
- Consider now Scheme B. When we have only two good modules left in the core, and if one of them fails, we still have one good module, but we don't know which one. If we arbitrarily choose one of them to continue, we will have a 50% chance of making the correct choice!



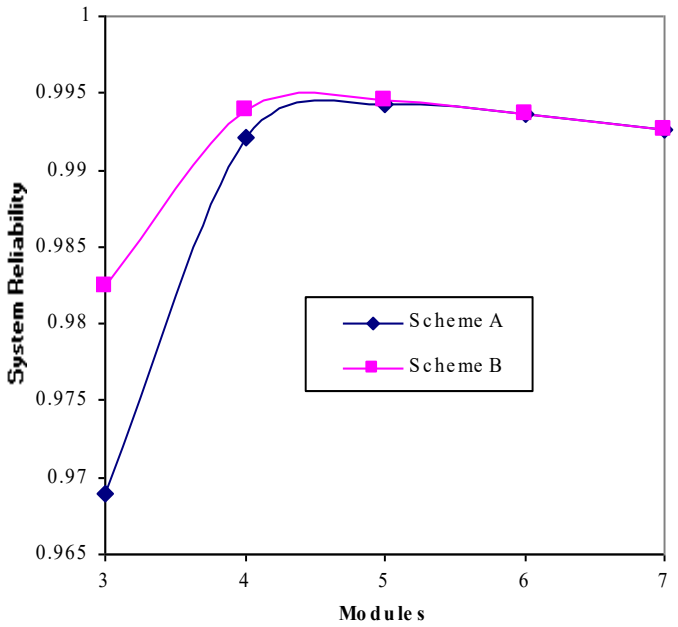
Scheme B is risky, but sometimes risk taking is needed for survival.

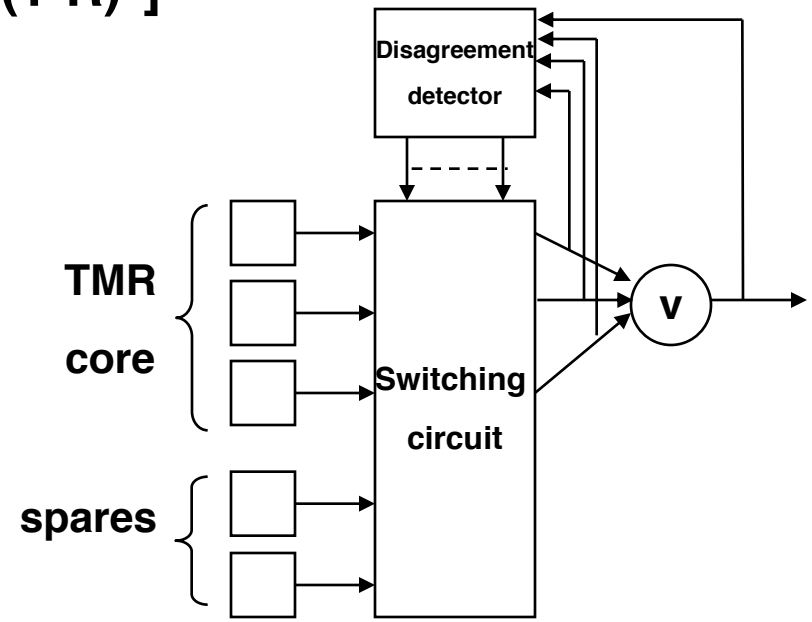
TMR+Spare: Scheme B

B: One of the last two fails, remove one arbitrarily.

$$3+2 \rightarrow 3+1 \rightarrow 3+0 \rightarrow 2+0 \rightarrow 1+0 \rightarrow \text{failure}$$

$$R_s = R^{an}\,[1-0.5nR(1-R)^{n-1}-(1-R)^n]$$





Variable Failure Rates

- We have assumed that the failure rate is constant. It greatly simplifies the calculations.
- It results in exponentially distributed time to failure.
- When the failure rate is varying, a generalization of the exponential distribution called Weibull distribution is used.
- The next slide is for your information only. For more details, consult the literature.

Generalization: Failure Rate

- $z(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} \quad where \beta \geq 0, \eta \geq 0$

  leads to Weibull Distribution

$$f_T(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} e^{-(\frac{t}{\eta})^{\beta}} \qquad R(t) = e^{-(\frac{t}{\eta})^{\beta}}$$

- β : shape parameter, 1 for constant rate
- η : scale parameter
- Often used to get a better fit, if needed, for rising or falling failure rates.

Weibull Distribution