

Fault Tolerant Computing

CS 530DL

Additional Lecture Notes

Modeling

Yashwant K. Malaiya
Colorado State University

Quantitative models

- **Derived from first principles:**
 - Arguments are actual things measured
 - Example: **monthly mortgage payment**

$$c = \frac{rP}{1 - (1 + r)^{-N}}$$

- **Empirical**
 - Arguments are just parameters
 - Example **Ideal body weight in kg = a+b (height in in. - c)**
Men: a=52 kg, b= 1.9, c = 60 in.
- **Combined**
 - Empirical with interpretation of parameters
 - From first principles, adjusted to fit

Derived using first principles

- **Obtain model**
 - Understand how things work
 - Use approximations
 - Derive formulas
- **Validate:**
 - Get real data
 - See if it fits. If not,
 - Make adjustments
 - Get alternative models
- **Use the model**
 - Plug in values to estimate

Empirical models

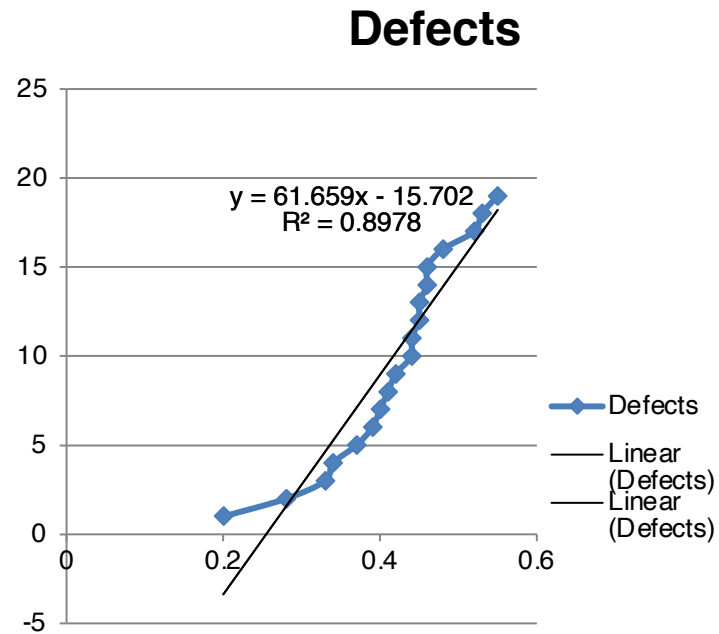
- **Look at data**
- **See if it resembles a function**
 - Linear, quadratic, logarithmic, exponential..
 - Involving 1, 2 or more parameters
- **See if it fits**
 - If not try something more complex
- **If it fits, see if an interpretation of the parameters is possible**
 - Not necessary, but will be good.

Curve fitting

- Get x-axis and y-axis numbers (x_i , y_i).
- Draw a scatter plot.
- Fit using Excel:
 - **linest()** or **logest()** functions
 - Select plot, rc, add **trendline**, select display options
 - Use **Solver** for general fitting
- Open Excel sheet Examples.xls

Example

Branch Cov	Defects
0.2	1
0.28	2
0.33	3
0.34	4
0.37	5
0.39	6
0.4	7
0.41	8
0.42	9
0.44	10
0.44	11
0.45	12
0.45	13
0.46	14
0.46	15
0.48	16
0.52	17
0.53	18
0.55	19



Curve fitting vs Predictive Capability

- A good model has good predictive capabilities.
- Curve fitting partial data may not necessarily identify the best model.

Curve Fitting

From

*Engineering Computation: An Introduction Using
MATLAB and Excel*

Chapter 5

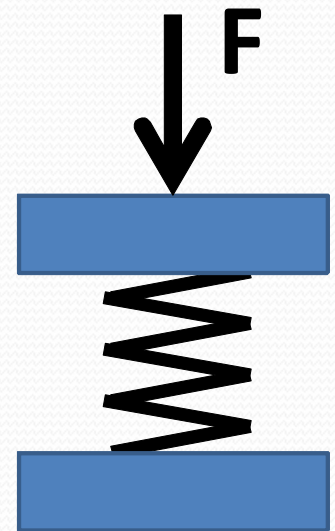


Curve Fitting

- Often, we have data points and we want to find an equation that “fits” the data
- Simplest equation is that of a straight line

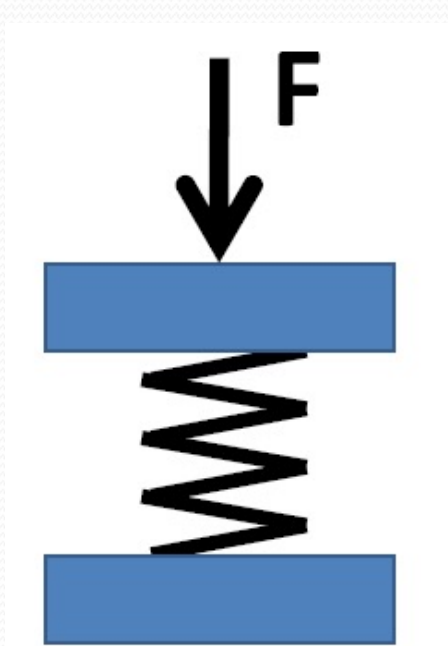
Curve Fitting Example

- A spring is placed between two flat plates and force is slowly applied to the upper plate, causing the spring to compress. When the force reaches a pre-load value of 25 N, the location of the upper plate is recorded. As the upper plate continues to move, the distance that the top plate moves, d , and the force required to move the plate, F , are recorded. As soon as the upper plate has moved 10 mm, the test is ended.



Curve Fitting Example

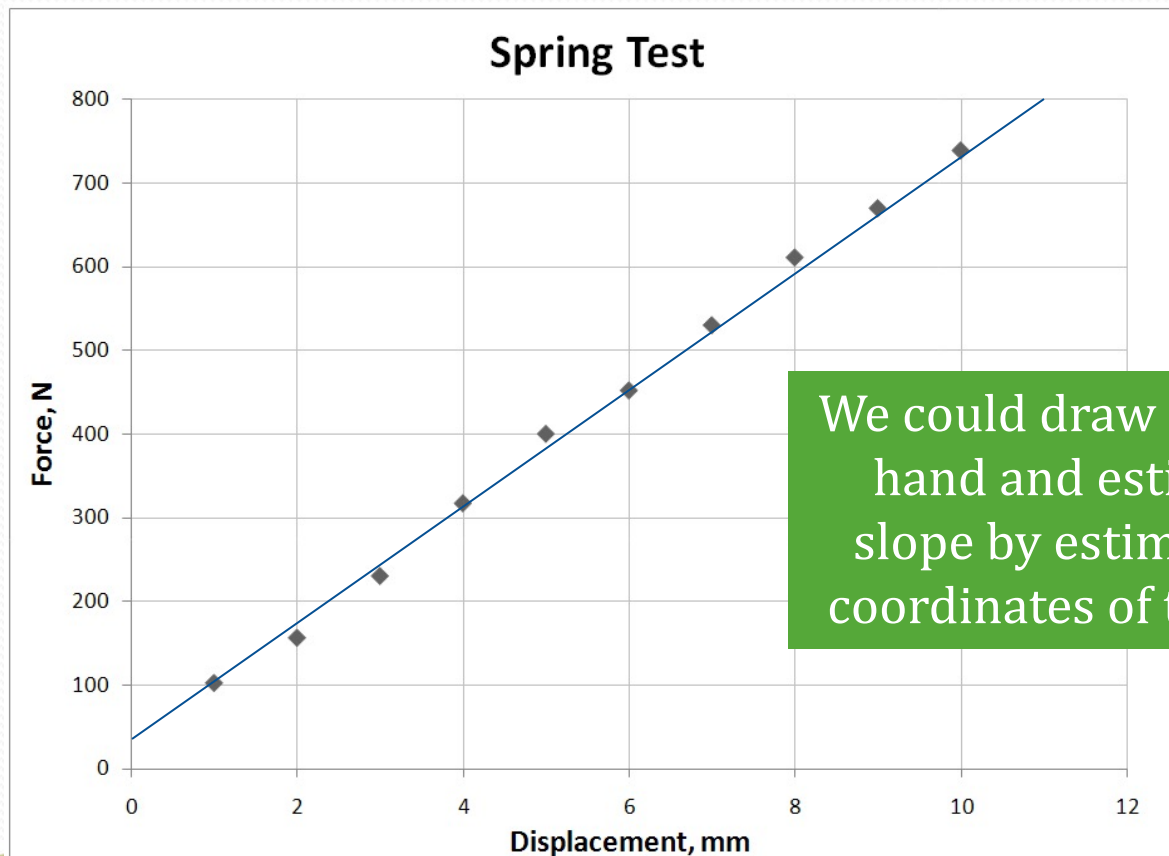
- Data from the test is shown here. The slope of the load-displacement curve is called the spring constant.



d, mm	F, N
1	102
2	156
3	230
4	317
5	400
6	452
7	530
8	611
9	670
10	739

Curve Fitting Example

- Data from a test – how do we find the slope?



We could draw in a line by hand and estimate its slope by estimating the coordinates of two points

Curve Fitting Example

- How do we find the line that *best* fits the data?
- Find the equation of the line that minimizes the sum of the *squares* of the differences between the values predicted from the equation and the actual data values
- Why do we square the differences? Because we are interested in the magnitudes of the differences. If one point is above the line and other is below it, we don't want these difference to cancel each other

Curve Fit Parameters

- In Excel, we can choose to have the equation of the best-fit line (the trendline) displayed in the form

$$y = mx + b$$

where m = the slope of the line

and b = the intercept of the line with the y-axis

Equation of Trendline

$$y = 72.28x + 23.13$$

- In this example, the slope of the line (the stiffness of the spring) equals 72.3 N/mm
- When the deflection (x) equals zero, the force (y) equals 23.1 pounds. This is consistent with our nominal pre-load of 25 pounds

Correlation Coefficient

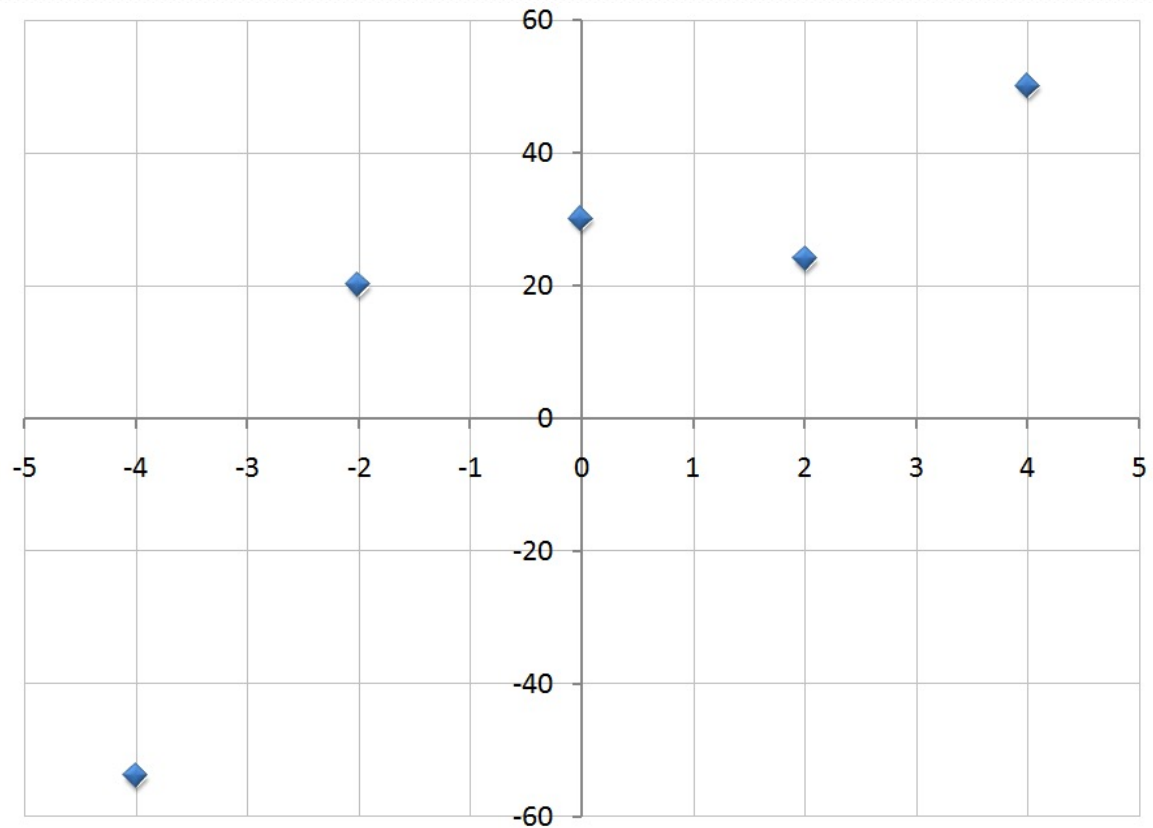
- The *correlation coefficient* (R^2) is a measure of how well the trendline fits the data
- A value of one represents a perfect fit
- In our example, the line fit is very good

$$y = 72.28x + 23.13$$
$$R^2 = 0.998$$

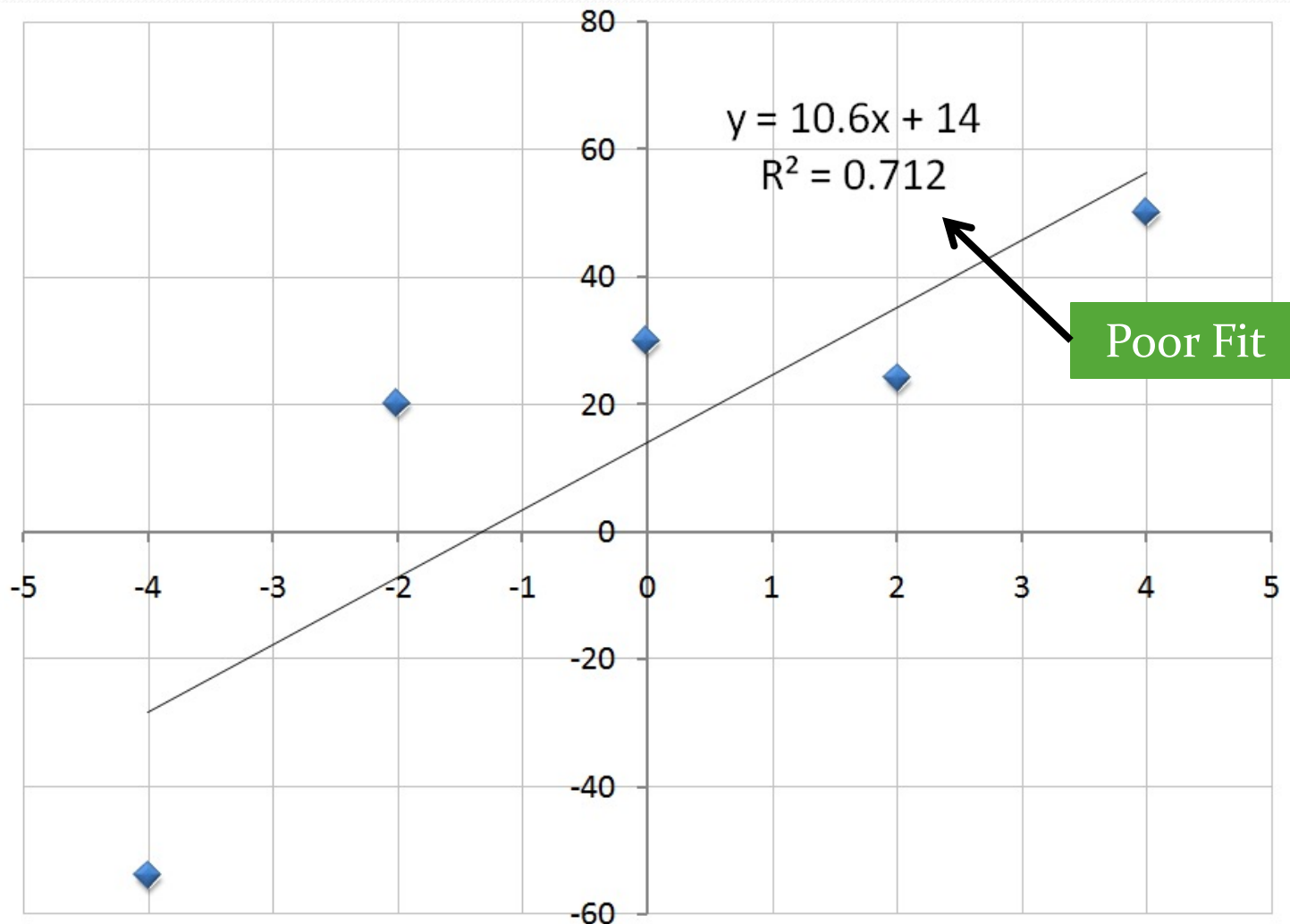
Curve Fitting Example

- Consider these five data points:

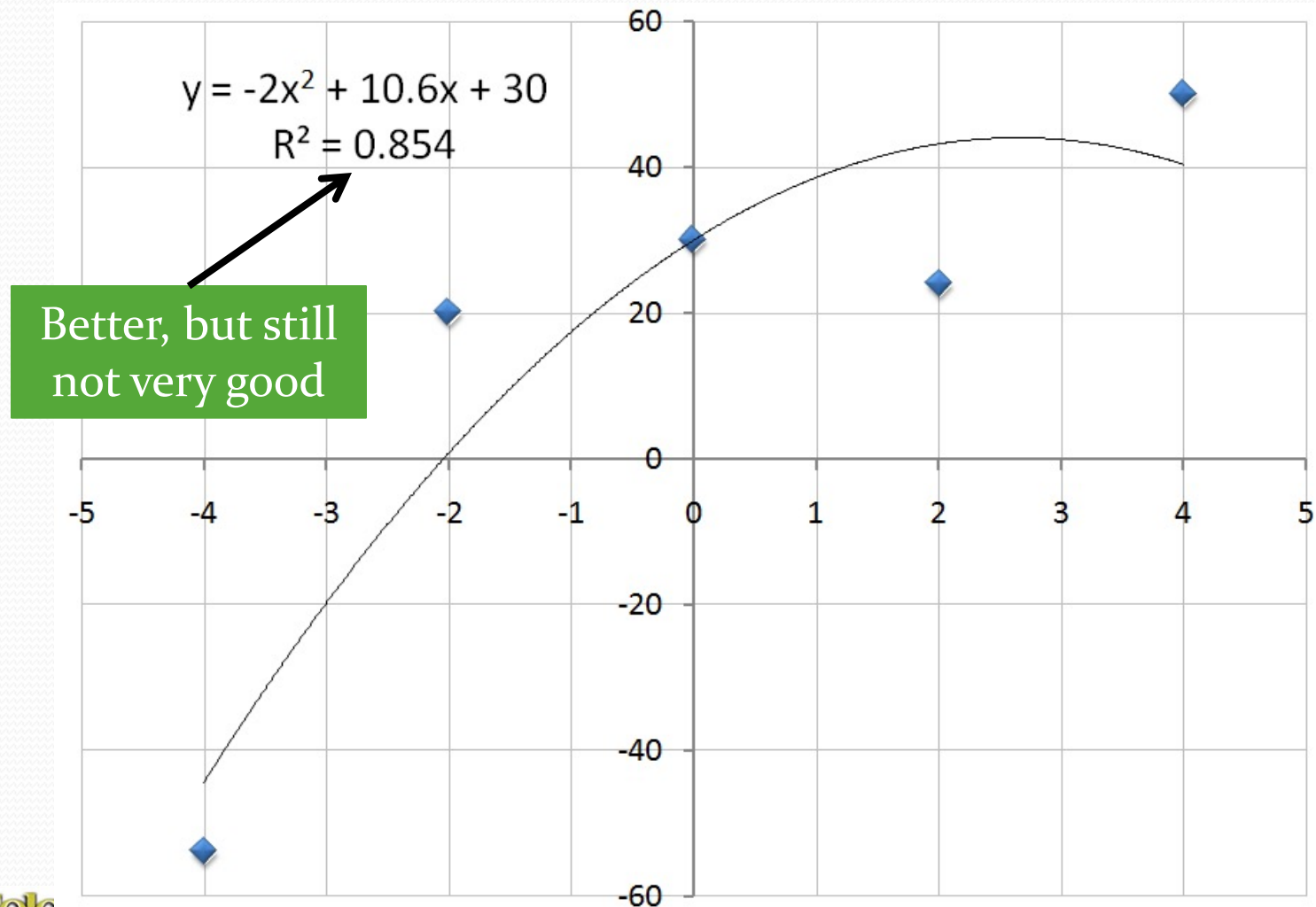
x	y
-4	-54
-2	20
0	30
2	24
4	50



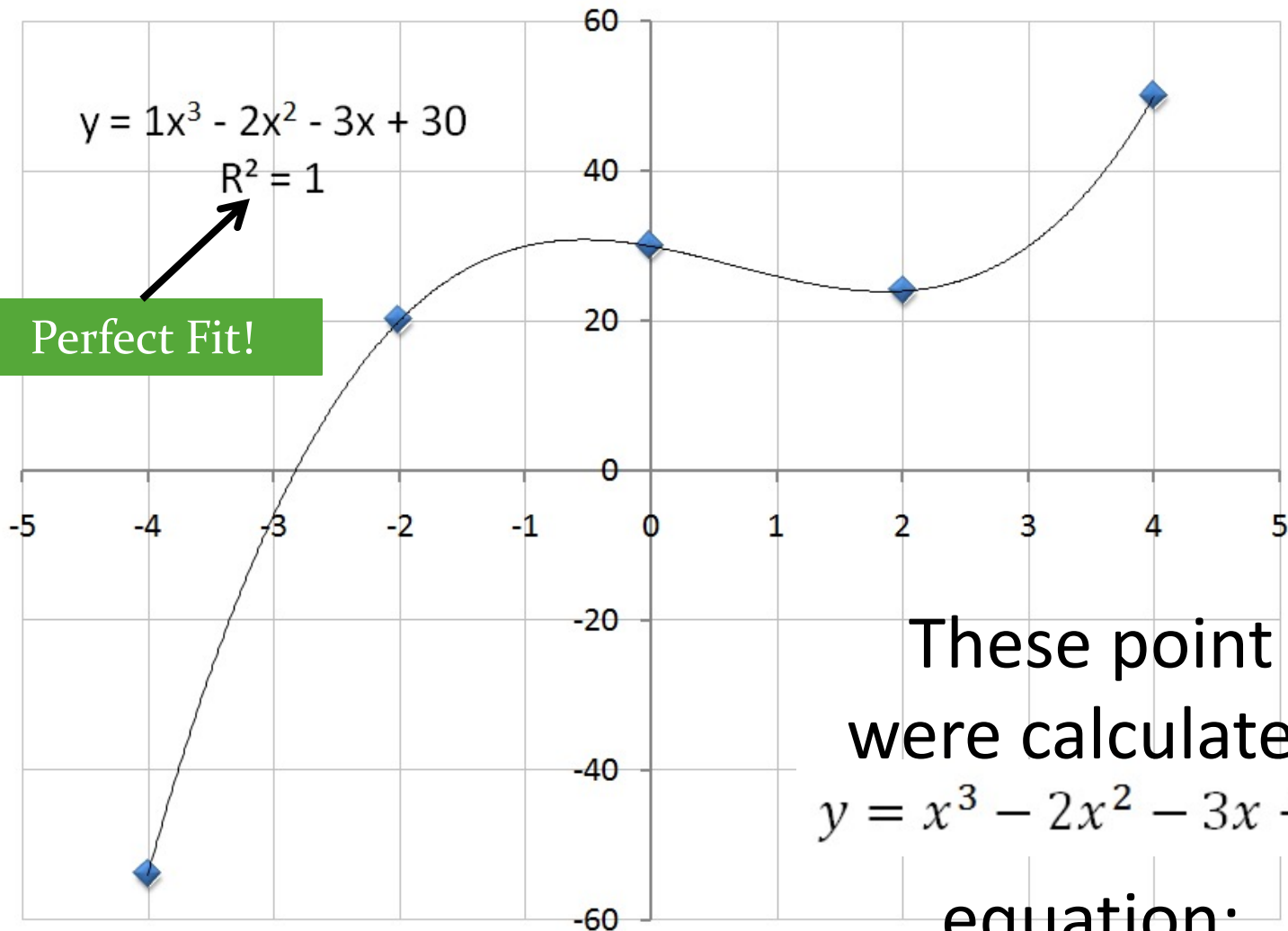
Linear Curve Fit



Try Second-Order Polynomial Fit



Third-Order Polynomial Fit



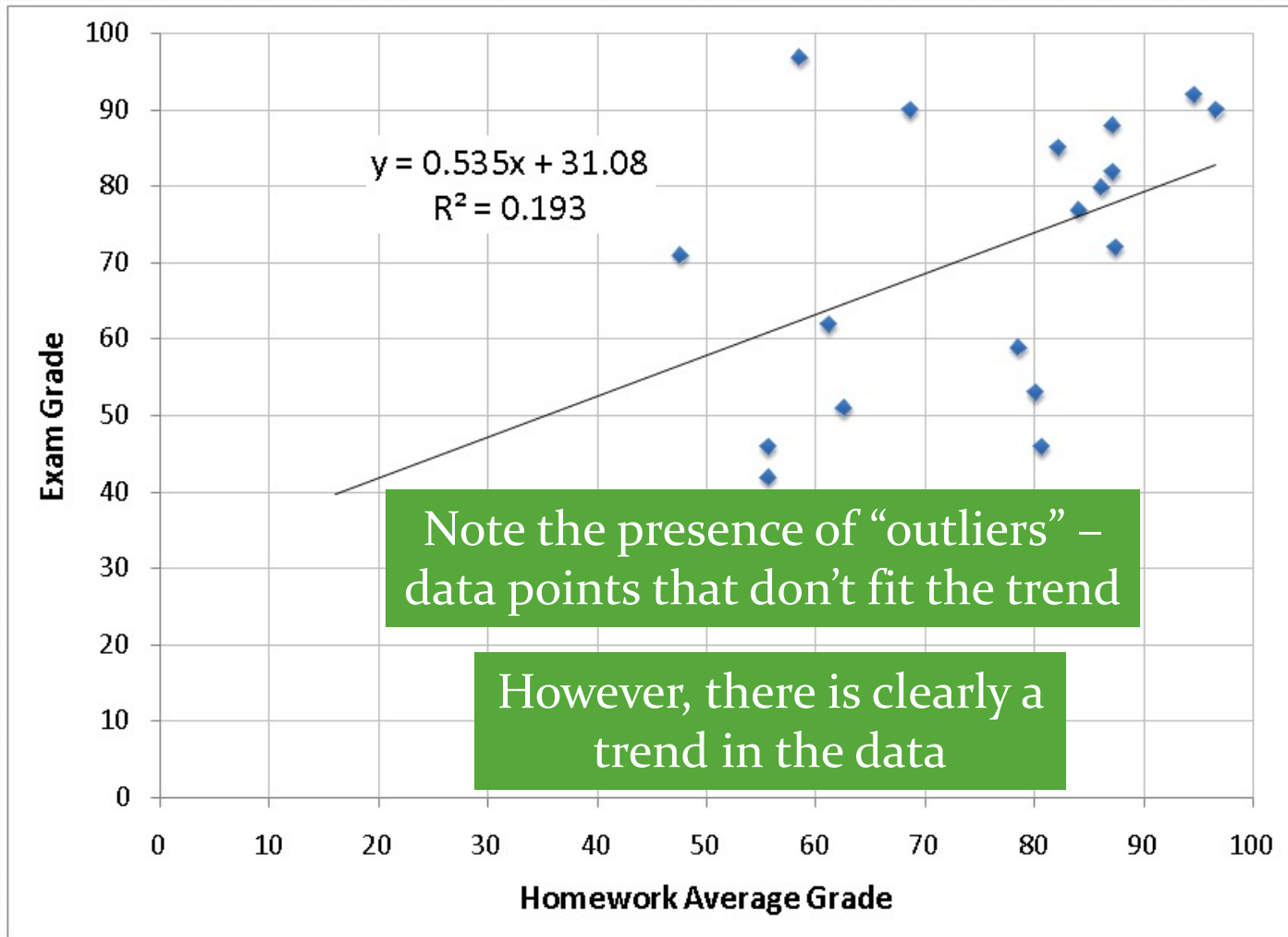
Perfect Fit!

These point
were calculated
 $y = x^3 - 2x^2 - 3x + 30$
equation:

Correlation Coefficient

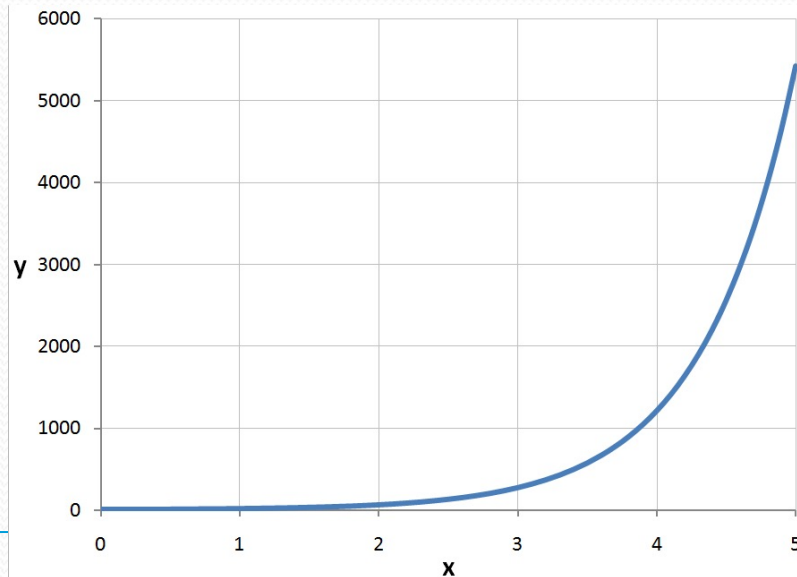
- A “good” fit to data is relative
- In the case of the spring example, the data should fit a mathematical model, and so an R^2 value of close to one is expected
- For other cases, a much lower R^2 value is expected
- Consider a comparison of final exam scores in a class vs. homework averages
- We would expect that students who do well on HW will generally do well on the final exam, but there will be exceptions

Exam vs. HW Example



Exponential and Power Equations

- Consider this equation: $y = 3e^{1.5x}$
- Here is a graph of the equation:



Exponential Equation

$$y = 3e^{1.5x}$$

$$\ln(y) = \ln(3e^{1.5x})$$

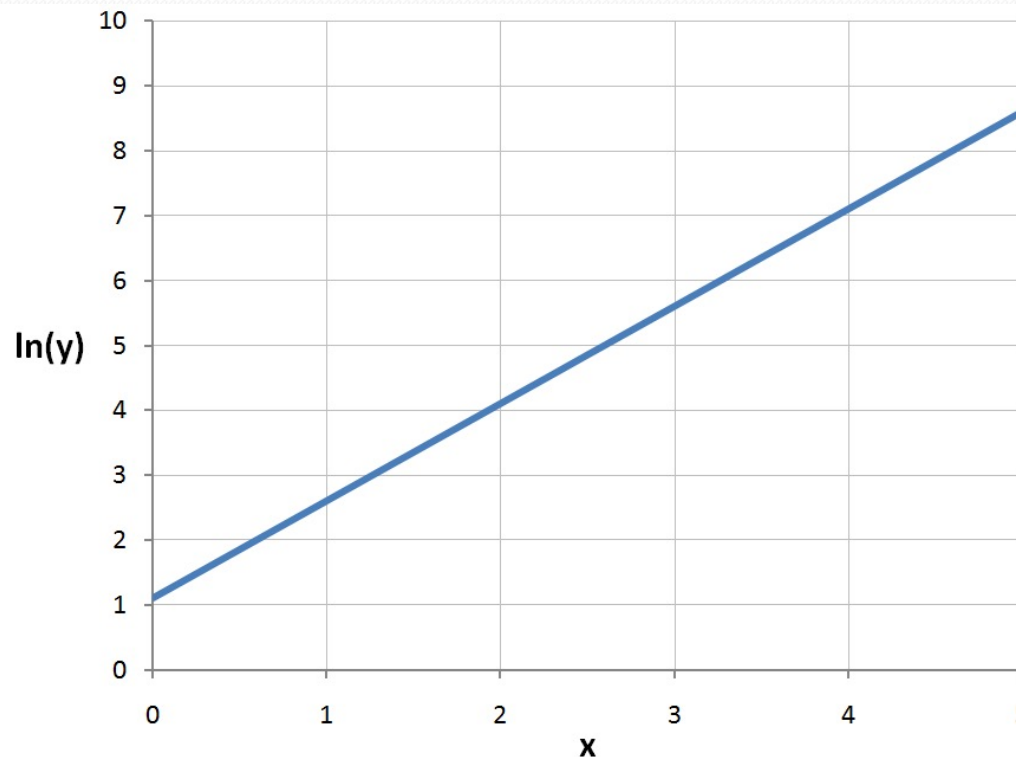
$$\ln(y) = \ln(3) + \ln(e^{1.5x})$$

$$\ln(y) = 1.5x + \ln(3)$$

If we plot the $\ln(y)$ instead of y , then we have the equation of a straight line

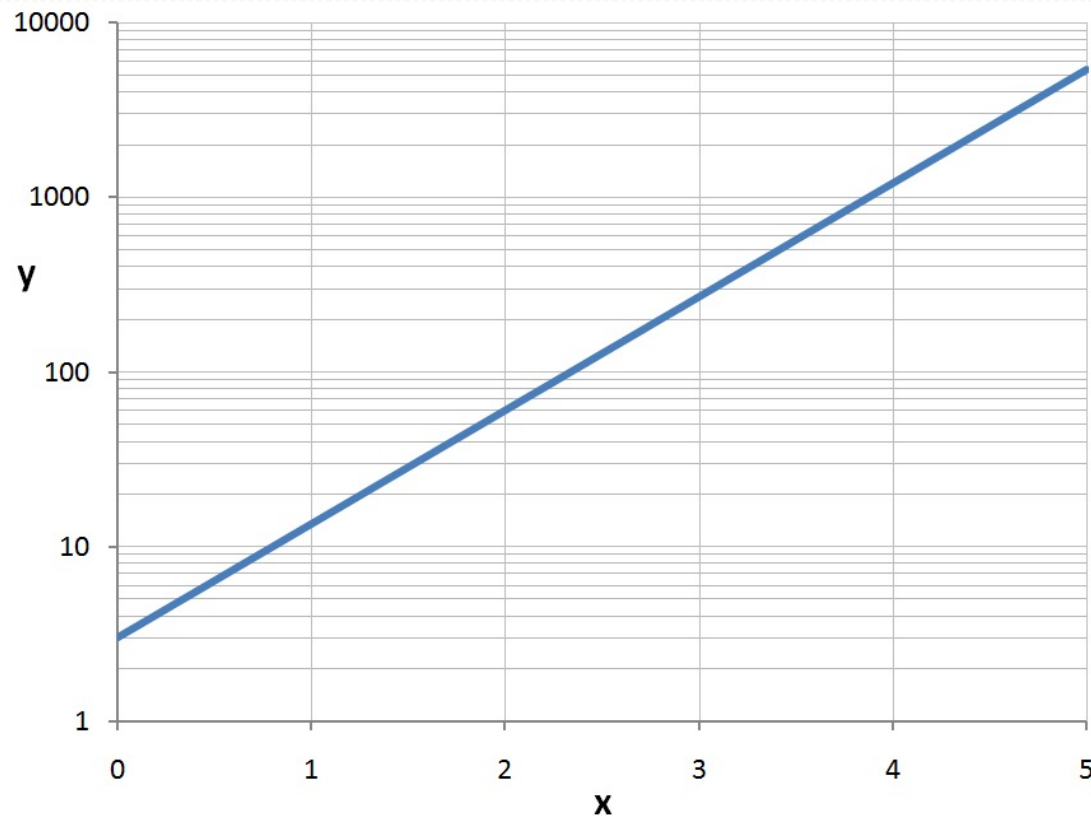
Exponential Equation

- This plot is not particularly useful, since it requires us to read the \ln of the dependent variable y , instead of y itself



Exponential Equation

- A better way is to display the y values on a *logarithmic scale*:



Exponential Equation

- We call this a *semi-log* plot since one of the axes is logarithmic
- Note that we have used a base-10 scale: remember that a logarithm can be converted to a logarithm of another base by multiplying by a constant:

$$\log_a(x) = \log_b(x) * \log_a(b)$$

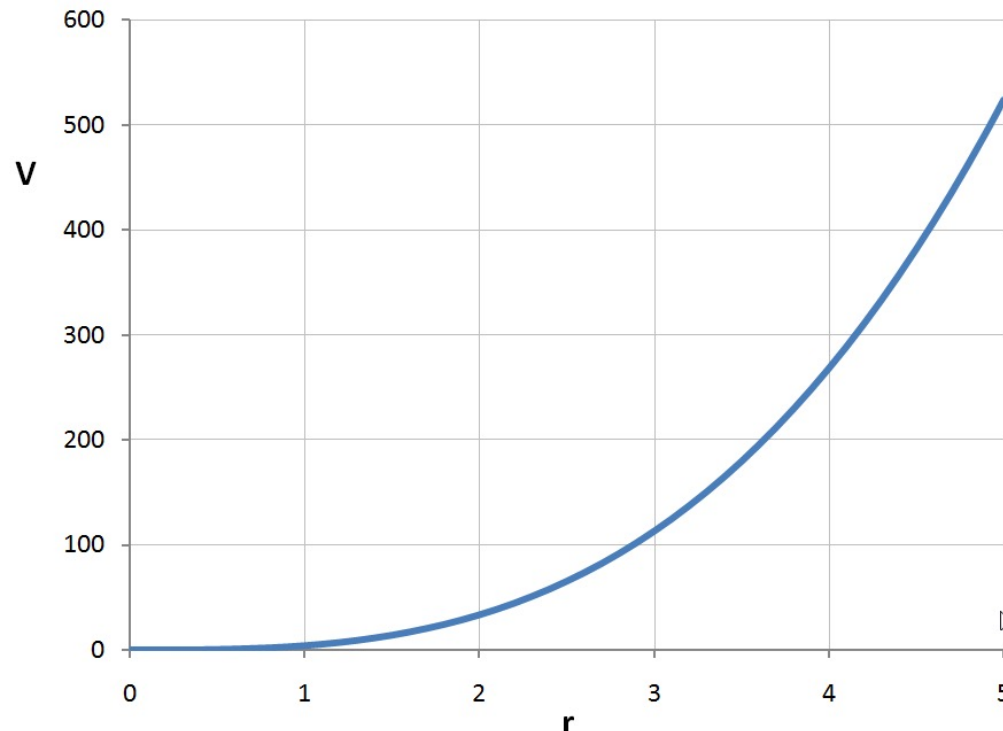
- Therefore, the equation will produce a straight line on semi-log axes regardless of the base of the logarithmic scale. Base 10 normally used – easiest to read
- Logarithmic scales can be specified in Excel and MATLAB

Power Equation

- Consider the equation for the volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

Plot:



Power Equations

$$V = \frac{4}{3} \pi r^3$$

$$\log(V) = \log\left(\frac{4}{3} \pi r^3\right)$$

$$\log(v) = \log\left(\frac{4}{3} \pi\right) + \log(r^3)$$

$$\log(V) = 3 * \log(r) + \log\left(\frac{4}{3} \pi\right)$$

If we plot the $\log(y)$ *and* $\log(x)$, then we have
the equation of a straight line

Power Equations

- This is called a *log-log* plot, since both axes are logarithmic

