Reliability of Multi-component Systems

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Virtual interaction

- I will ask some questions during the video lecture only.
  - You will note down the question number and your response on paper and save it.
  - You will need to use your response in on-line quizzes, when the answer will be evaluated.

Software Reliability: Review

We have discussed

- Defect density
- Testing
  - Exponential and other SRGMs
    - Software size and parameter values
  - Failure intensity, fault exposure ratio

Reliability of Multi-component Systems

- Software system: number of modules.
- Individual modules developed and tested differently: different defect densities and failure rates.
  - **Sequential execution**
  - **Concurrent execution**
  - **N-version systems**

Reliability of Multi-component Systems

- Definition of reliability? Possible Perspectives:
  - System is reliable if there is no defect in any of the components.
    - Deterministic, not really useful.
  - Probability of a transaction running without a failure.
    - Depends on how often the failure is encountered.
  - If the system fails, what is the impact of the failure?
    - Risk due to a failure = its probability x its impact

Sometimes a transaction may be rerun after a failure. That is time redundancy.

Sequential execution

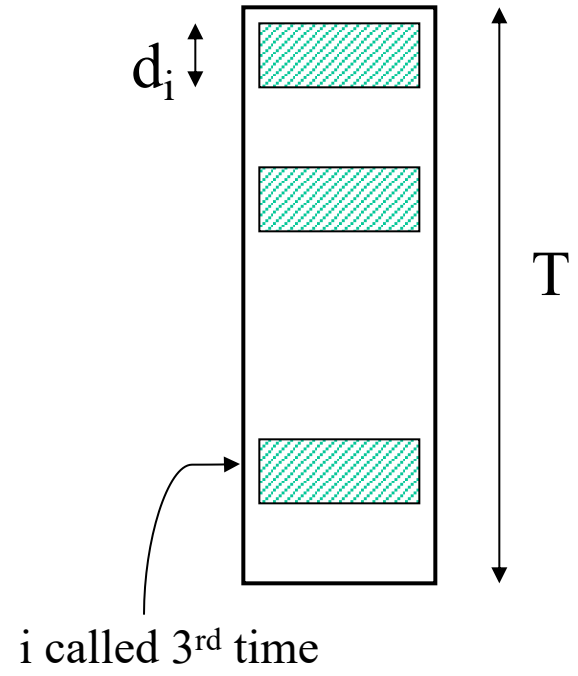
- Traditional sequential programs:
  - Assume one module executed at a time.
  - f_i: fraction of time module i under execution
  - λ_i its failure rate
- Mean system failure rate:

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \lambda_i$$

Sequential Execution (cont.)

- T: mean duration of a single transaction
- module i is called e_i times during T, each time executed for duration d_i

$$f_i = \frac{e_i \ d_i}{T}$$



Sequential Execution (cont.)

- System transaction reliability $R_{sys} = \exp(-\lambda_{sys} T)$

$$R_{sys} = \exp\left(-\sum_{i=1}^{n} e_i \, d_i \, \lambda_i\right)$$

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \, \lambda_i$$

- Since $\exp(-d_i\lambda_i)$ is R_i,

$$R_{sys} = \prod_{i=1}^{n} (R_i)^{e_i}$$

$$f_i = \frac{e_i \, d_i}{T}$$

Sequential Execution

Reliability $e^{-\lambda T}$ Pr(failure in T) = $1 - e^{-\lambda T}$

	Called times	Av duration	Fraction of T	Failure rate	Av failure rate/unit time	Rsys
Module i	ei	di	fi	λi	λsys	
a	1	3	12%	0.01		
b	2	4	32%	0.03		
c	7	2	56%	0.001		
	Total time T	25	100%		0.0114	**0.7527**

Sequential Execution Risk

- System Risk = Σ Risk due to failure type I

For a specific time frame, assuming

- $Risk_i = \Pr(failure\ type\ i) \times penalty\ for\ i$
  - $\Pr(failure\ type\ i) = 1 - e^{\lambda_i f_i T} i$
  - $Risk_i = (1 - e^{\lambda_i f_i T} i) \times P_i$
  - System Risk = $\sum_{i=1}^{n} (1 - e^{\lambda_i f_i T} i) \times P_i$

Sequential Execution Risk

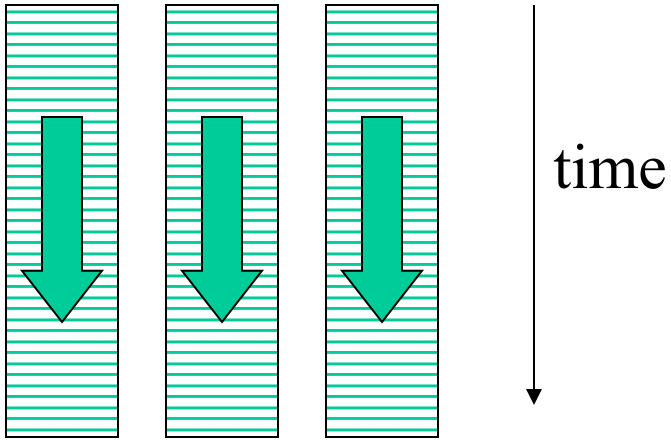
- System Risk = Σ Risk due to failure type i

	Called times	Av duration	Fraction of T	Failure rate	Av cost/ failure	Fail prob/T	Risksi
Module i	ei	di	fi = di/T	λi	Ci	=1-exp(ei.di. λi)	potential loss per trans
a	1	3	12%	0.01	20	0.030	0.59
b	2	4	32%	0.03	100	0.213	21.34
c	7	2	56%	0.001	200	0.014	2.78
Total time	T	25	100%			**Total risk**	**24.71**

Concurrent execution

Concurrently executing modules, within a system or in a distributed system.

- all concurrent modules must run without failures for system to function.
- j concurrently executing modules

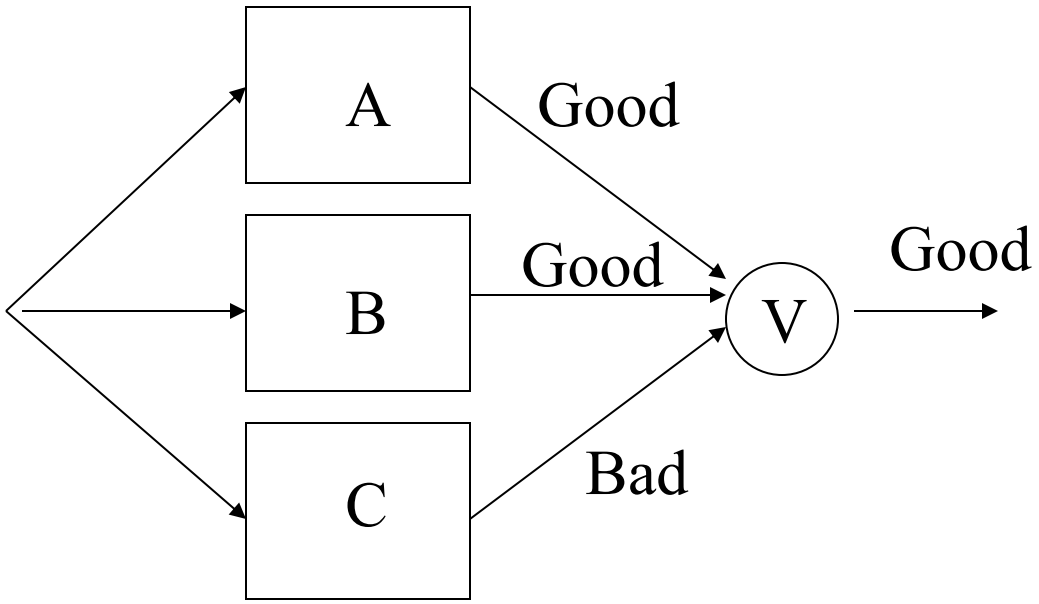


$$\lambda_{sys} = \sum_{j=1}^{m} \lambda_j$$

N-version systems

- Critical applications, like defense or avionics
- Each version is implemented and tested independently
- Common implementation uses triplication and voting on the result

N-version Systems (Cont.)



$R_{sys}=1-(1-R)^3-3R(1-R)^2$

$R=0.9 \Rightarrow R_{sys}=.972$

$\bar{R}=0.1 \Rightarrow \bar{R}_{sys}=.028$

N-version systems: Correlation

- Correlation significantly degrades fault tolerance
- Significant correlation common in N-version (Knight-Leveson)
- Is it cost effective?

N-version systems: Correlation

- 3-version system
- q_3: probability of all three versions failing for the same input.
- q_2: probability that any two versions will fail together.
- Probability P_{sys} of the system *failing* for a transaction

$$P_{sys} = q_3 + 3q_2$$

N-version systems: Correlation

- Example: *data collected by Knight-Leveson; computations by Hatton*
- *3-version system, probability of a version failing for a transaction 0.0004*
- *in the absence of any correlated failures*

$$P_{sys} = (0.0004)^3 + 3(1 - 0.0004)(0.0004)^2$$

$$= 4.8 \times 10^{-7}$$

- Uncorrelated improvement factor of $0.0004/4.8 \times 10^{-7} = 833.3$

Les Hatton. 1997. N-Version Design Versus One Good Version. IEEE Software, 14, 6 (November 1997), 71-76.
J. C. Knight, N. G. Leveson and L. D. S. Jean, "A Large Scale Experiment in N-Version Programming", in 15th Int. Symp. on Fault Tolerant Computing (FTCS-15), pp.135-139, IEEE Computer Society Press, 1985

N-version systems: Correlation

$$P_{sys} = q_3 + 3q_2$$

- Uncorrelated improvement factor of $0.0004/4.8 \times 10^{-7} = 833.3$
- Correlated: $q_3 = 2.5\times10^{-7}$ and $q_2 = 2.5\times10^{-6}$
- $P_{sys} = 2.5\times10^{-7} + 3\times2.5\times10^{-6} = 7.75\times10^{-6}$
- improvement factor: $0.0004/7.75\times10^{-6} =$ **51.6**
- state-of-the-art techniques can reduce defect density only by a factor of **10!**
- Thus 3-version system may be worth considering in some cases.