

Quantitative Cyber-Security

Colorado State University

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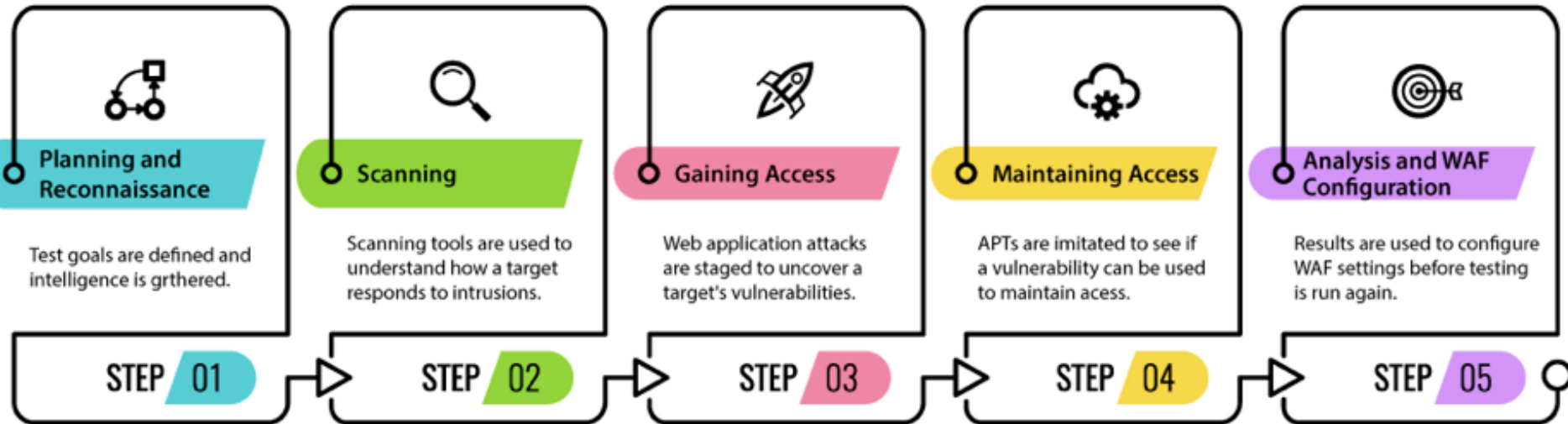
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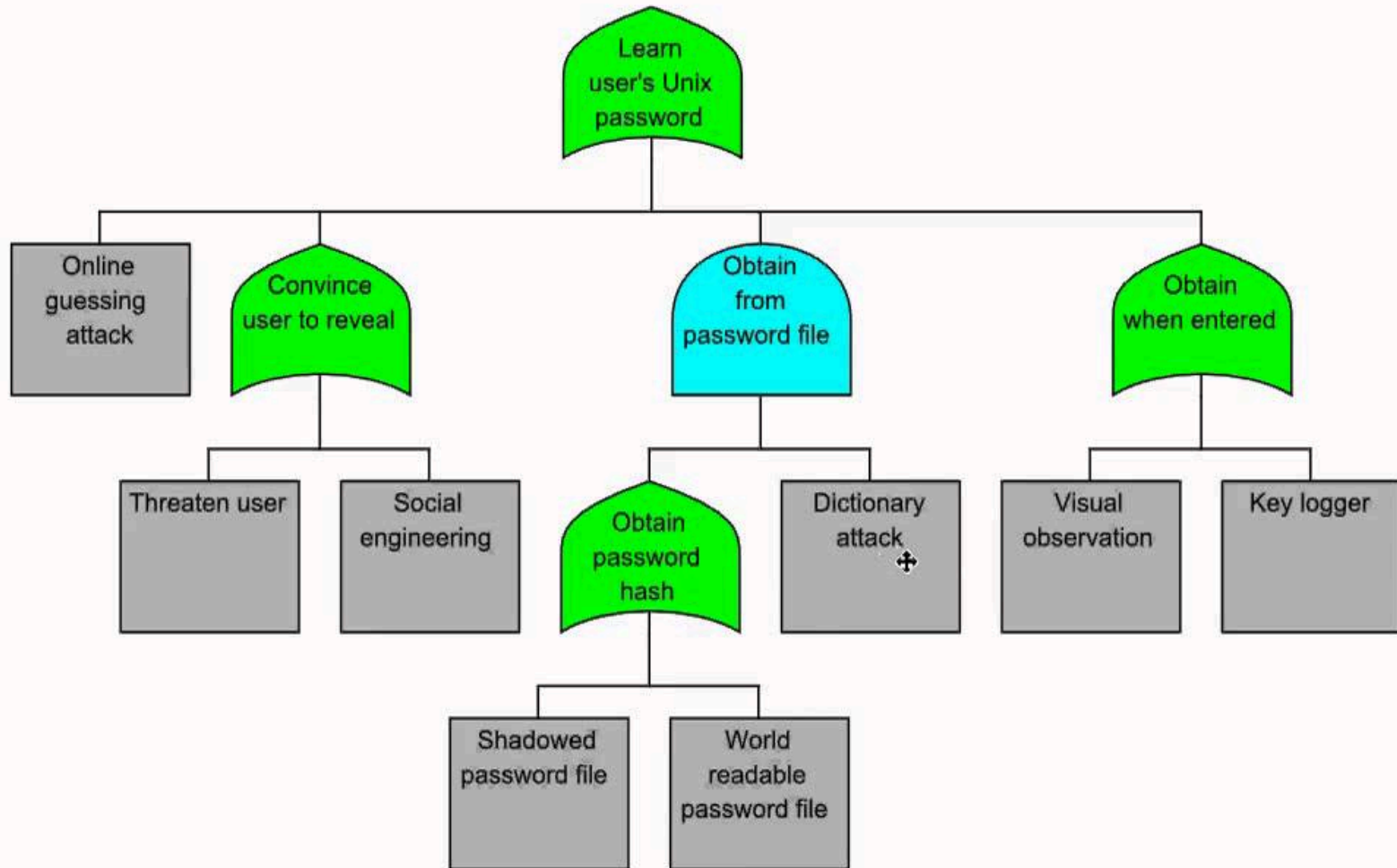
Pen Testing Stages



1. Planning and reconnaissance
2. Scanning
3. Gaining access
4. Maintaining access:
5. Analysis and remediation

Sources: [1](#), [2](#)

Attacks and Attack trees



Topics

- Risk components
- Probability of a breach
- Gordon-Loeb Models

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Breach probability



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Risk as a composite measure

Formal definition:

- **Risk** due to an adverse event e_i
$$\text{Risk}_i = \text{Likelihood}_i \times \text{Impact}_i$$
- Likelihood $_i$ may be replaced by frequency $_i$, when it may happen multiple times a year.
- This yields the expected value. Sometimes a worst-case evaluation is needed.

In classical risk literature, the internal component of Likelihood is termed “Vulnerability” and external “Threat”. Both are probabilities. There the term “vulnerability” does not mean a security bug, as in computer security.

Risk as a composite measure

- Likelihood can be split in two factors

$$\text{Likelihood}_i = P\{\text{A security hole}_i \text{ is exploited}\}.$$

$$= P\{\text{hole}_i \text{ present}\}.$$

$$P\{\text{exploitation} \mid \text{hole}_i \text{ present}\}$$

- $P\{\text{hole}_i \text{ present}\}$: an **internal** attribute of the system.
- $P\{\text{exploitation} \mid \text{hole}_i \text{ present}\}$: depends on circumstances **outside** the system, including the adversary capabilities and motivation.
- In the literature, the terminology can be inconsistent.

Caution: In classical risk literature, the internal component of Likelihood is termed “**Vulnerability**” and external “**Threat**”. Both are probabilities. There the term “vulnerability” does not mean a security bug, as in computer security.

Annual Loss Expectancy (ALE)

Note the terminology is from the Risk literature.

- Annual loss expectancy (ALE). (It is a risk measure)

$$\text{ALE} = \text{SLE} \times \text{ARO}$$

– Where ARO is Annualized rate of occurrence.

- A countermeasure reduces the ALE by reducing one of its factors.

COUNTERMEASURE_VALUE

$$= (\text{ALE_PREVIOUS} - \text{ALE_NOW}) - \text{COUNTERMEASURE_COST}$$

ALE_PREVIOUS: ALE before implementing the countermeasure.

ALE_NOW: ALE after implementing the countermeasure

COUNTERMEASURE_COST: *annualized* cost of countermeasure

Estimating the Breach Probability

What factors impact the probability of an organization to be breached?

- **Breach size**
- **Other factors:**
- **Default value of factor = 1**
 - Specific value relative to the default value
- **Factors based on available data**
 - Organization's Country F_{country}
 - Organization's Industry Classification F_{industry}
 - Sensitive Data Encryption $F_{\text{encryption}}$
 - Organization's Privacy F_{privacy}
 - Business Continuity Management Team F_{BCM}
 - Data Breach Causes $F_{\text{breach_cause}}$

Modeling the Breach Probability

What factors impact the probability of an organization to be breached?

- Breach size
- Other factors:
- Default value of factor = 1
 - Specific value relative to the default value
- Do factors add or multiply?
 - Factors largely orthogonal: multiplicative
 - Factors overlap: additive
- Examples of multiplicative models
 - COCOMO Cost estimation model
 - RADC software defect density model
 - VLSI failure rate models

Breach Probability Model

A proposed model for the probability of a breach for the next

$$P \{\text{breach}\} = F_{country} * F_{BCM} * F_{industry} * \\ F_{breach_{cause}} * F_{encryption} * F_{privacy} * \\ \alpha \exp(-\beta x)$$

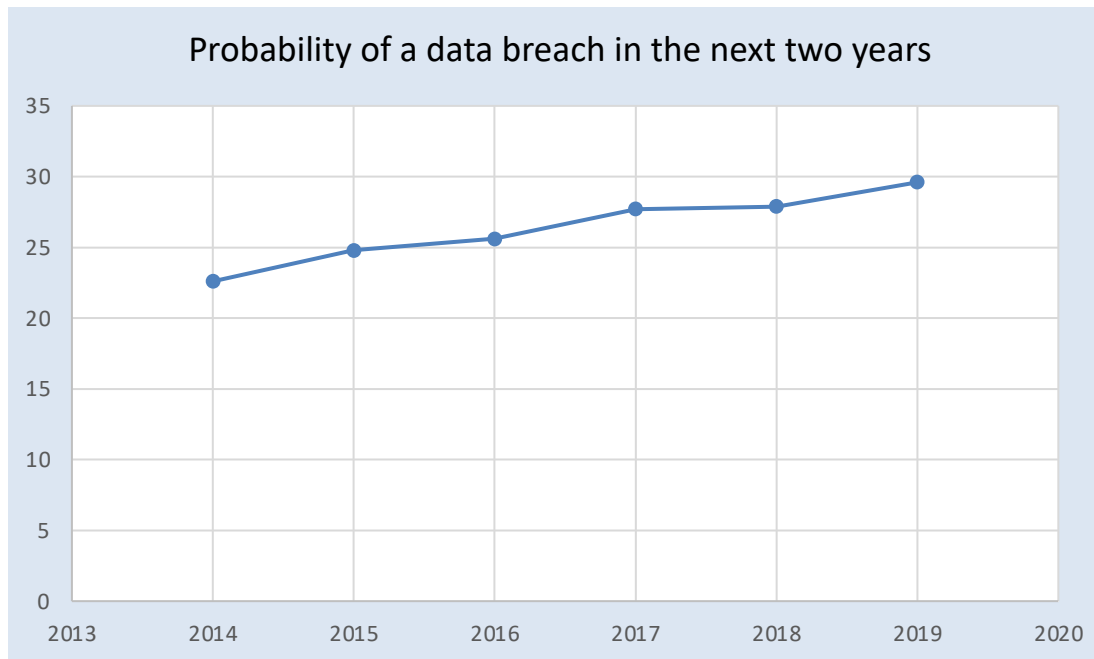
Where $\alpha = 0.4405$, $\beta = 4E-05$, x the breach size 2015

Justification in the following slides.

Data Breach Probability

[Cost of a Data Breach Report 2019](#), IBM Security, study by Ponemon Institute.

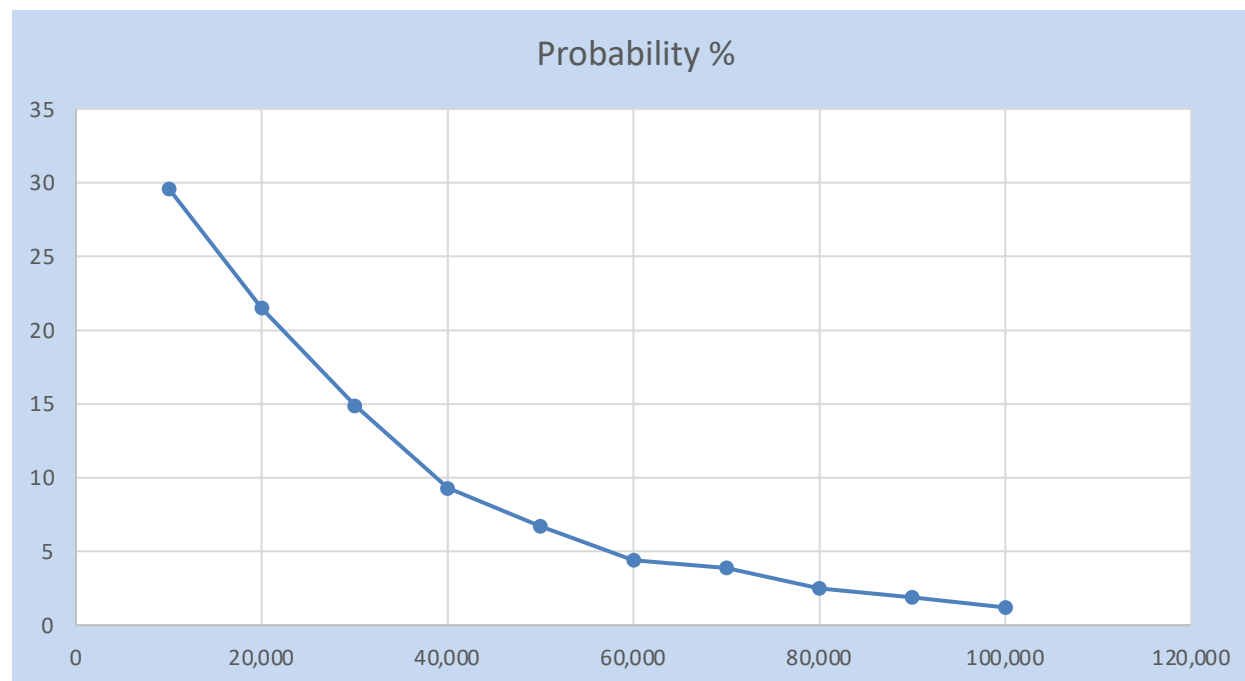
- 507 participating companies, with a minimum of 10,000 records
- United States, India, the United Kingdom, Germany, Brazil, Japan, France, the Middle East, Canada, Italy, South Korea, Australia, Turkey, ASEAN, South Africa, Scandinavia



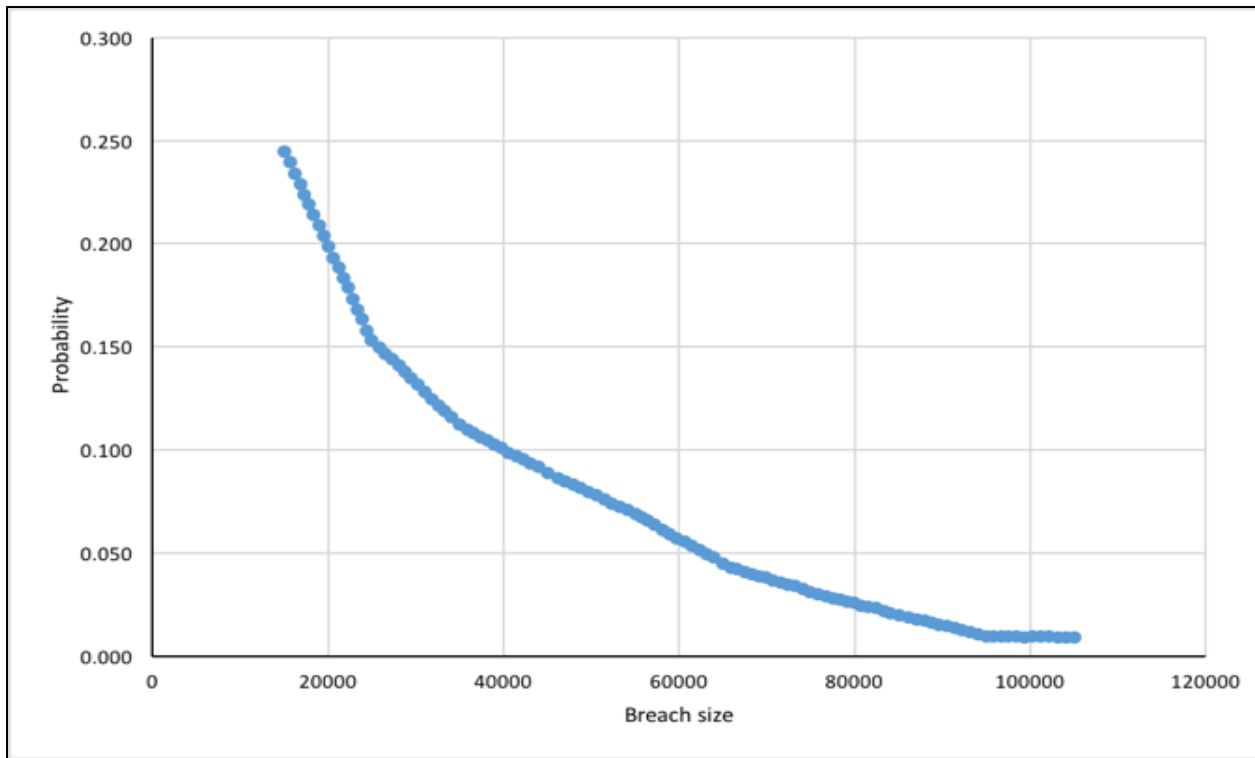
Probability of a data breach by number of records lost

Over the next two years, involving minimum of 10,000 and maximum of 100,000 records.

[Cost of a Data Breach Report 2019](#), IBM Security, study conducted by Ponemon Institute.

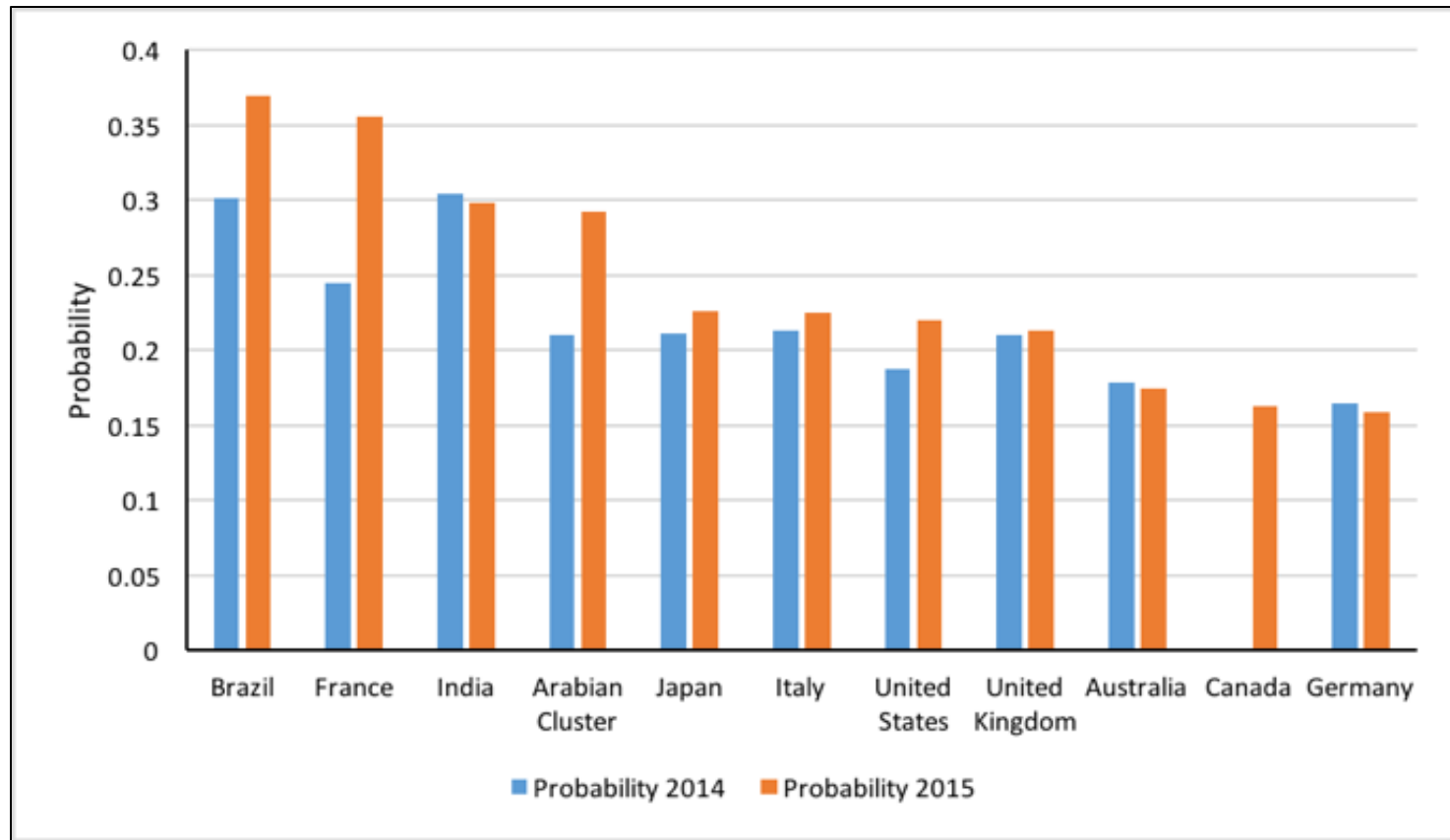


Breach probability -Breach size



Data breach probability for the next two years based on the breach size (Ponemon data 2015)

Data breach probability by country



Data breach probability by country (Ponemon data 2015)

A minimum of 10,000 compromised records

Data breach probability by country

Data breach probability by country Fcountry (Ponemon data 2015)

Country Name	Multiplier
USA	1 (default)
Germany	0.72
Canada	0.74
France	1.62
UK	0.97
Italy	1.02
Japan	1.03
Australia	0.79
Arabian Cluster (Saudi Arabia and United Arab Emirates)	1.33
Brazil	1.68
India	1.35

Organization's Industry Classification Findustry

Model proposed:

Industry classification	Multiplier
Communications	1.09
Consumer Products	1.24
Education	1.30
Financial Services	0.98
Government Services	1.63
Healthcare and Pharmaceuticals	1.26
Industrial	0.77
Retail	1.69
Services: professional and general services	1.48
Technology and software	1.26
Transportation	0.86
All others	1 (default)

Business Continuity Management Team FBCM

Model proposed:

BCM involved in incident response plan	Multiplier
Yes	0.84
No	1.1
Not sure	1 (default)

Sensitive Data Encryption Fencryption

Model proposed:

Encryption of sensitive data	Multiplier
Yes	0.64
No	1.03
Not sure	1 (default)

Organization's Privacy Fprivacy

Model proposed:

Organization's Privacy	Multiplier
A formal privacy and data protection program that is enterprise-wide	0.89
A formal privacy and data protection program that is not enterprise-wide	0.92
An informal privacy and data protection program that is enterprise-wide	1 (default)
An informal privacy and data protection program that is not enterprise-wide	1.19
No privacy or data protection program in place	1.42

Data Breach Causes Fbreach_cause

Model proposed:

Data breach causes	Multiplier
Malicious or criminal attack	1.32
Negligence or mistakes (Human error)	0.82
System glitch	0.75
don't know	1 (default)

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Gordon-Loeb Models



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L. A. Gordon and M. P. Loeb, “The economics of information security investment,” *ACM Trans. Inf. Syst. Secur.*, vol. 5, no. 4, pp. 438–457, 2002.

Gorden Loeb models

- L. A. Gordon and M. P. Loeb, “The economics of information security investment,” *ACM Trans. Inf. Syst. Secur.*, vol. 5, no. 4, pp. 438–457, 2002.
- Model for the impact of a security investment on the probability of a breach.
 - $S(z,v)$
 - S : probability of a breach after an investment z
 - v : probability of a breach before investment
- Derived using concepts from economics, without using any data.
- Further work needed.

Security breach probability function

Security breach probability function. $S(z, v)$

- where $z > 0$ denote the monetary (e.g., dollar) investment in security to protect the given information set.
- $v =$ “vulnerability” (probability of a security breach before investment)

Assumptions concerning $S(z, v)$:

A1. $S(z, 0) = 0$ for all z . If the information is completely invulnerable, then it will remain perfectly protected for with a zero investment.

A2. For all v , $S(0, v) = v$. That is, if there is no investment in information security, the probability of a security breach, conditioned on the realization of a threat, is the inherent vulnerability, v .

A3. For all $v \in (0, 1)$, and all z , $S_z(z, v) < 0$ and $S_{zz}(z, v) > 0$, where S_z denotes the partial derivative with respect to z and S_{zz} denotes the partial derivative of S_z with respect to z .

That is, as the investment in security increases, the information is made more secure, but at a decreasing rate. Furthermore, we assume that for all $v \in (0, 1)$, $\lim_{z \rightarrow \infty} S(z, v) \rightarrow 0$, so by investing sufficiently in security, the probability of a security breach, t times $S(z, v)$, can be made to be arbitrarily close to zero.

Impact of investment z :

The **expected benefits of an investment in information security**, $EBIS$, are equal to the reduction in the firm's expected loss attributable to the extra security.

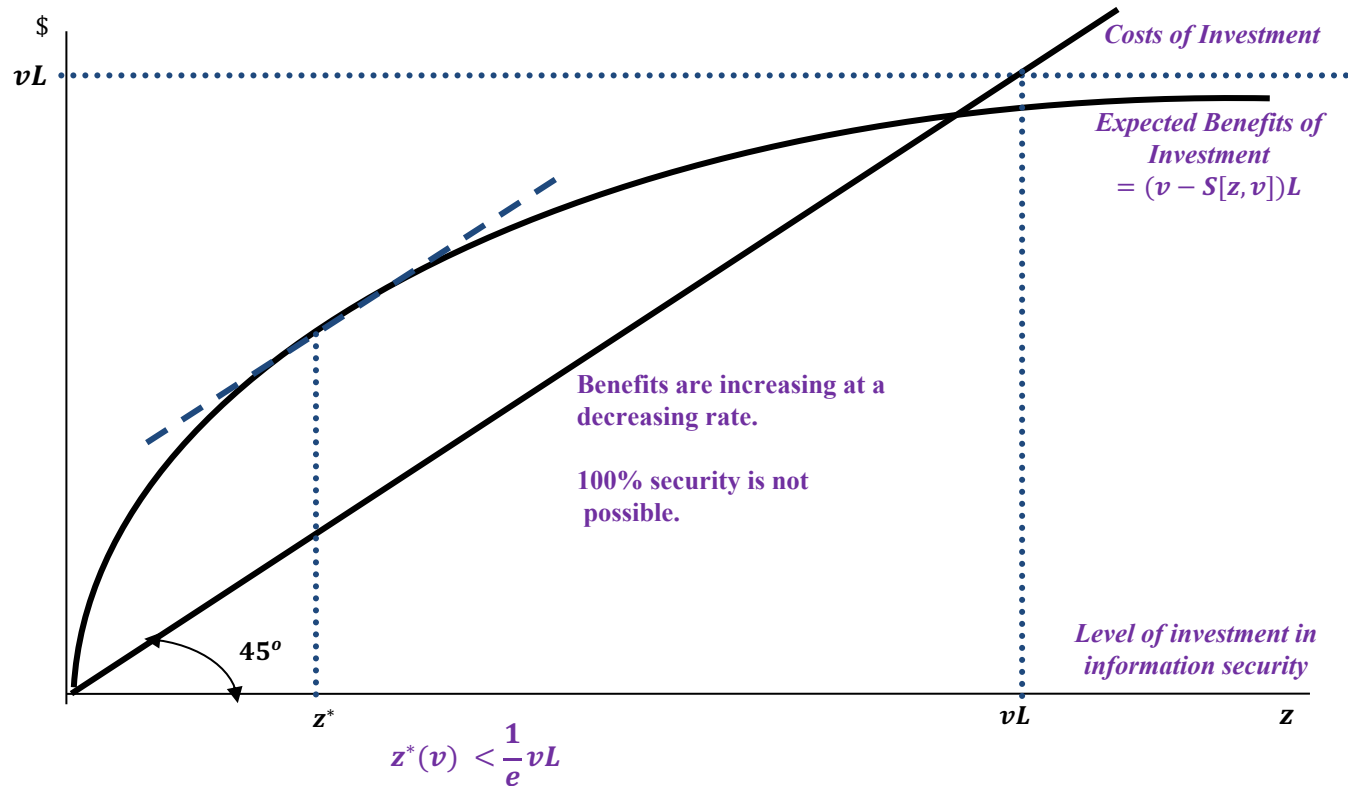
$$EBIS(z) = [v - S(z, v)] L$$

The **expected net benefits from an investment in information security**, $ENBIS$ equal $EBIS$ less the cost of the investment, or:

$$ENBIS(z) = [v - S(z, v)] L - z$$

v – Probability of security breach
 L – Potential Loss. vL – Expected Loss
 z – Level of Investment
 $S[z, v]$ – Revised probability of breach

Benefits & Costs of an Investment in Cyber/Information Security



- v – Vulnerability (Probability of security breach)
- L – Potential Loss
- vL – Expected Loss
- z – Level of Investment
- z^* – Optimal Investment Level
- $S[z, v]$ – Revised v after z (Revised probability of breach)

Security breach probability functions

They proposed two broad classes of security breach probability functions that *satisfy A1-A3*.

- The first class of security breach probability functions, denoted by $S^I(z, v)$, is given by:

$$S^I(z, v) = \frac{v}{(\alpha z + 1)^\beta}$$

where the parameters $\alpha > 0$, $\beta \geq 1$ are measures of the **productivity of information security** (i.e., for a given (v, z) , the probability of a security breach is decreasing in both α and β).

Solving for optimal z^*

$$z^{I^*}(v) = \frac{(v\beta\alpha L)^{1/(\beta+1)} - 1}{\alpha}.$$

v – Probability of security breach
 L – Potential Loss. vL – Expected Loss
 z – Level of Investment
 $S[z, v]$ – Revised probability of breach

Security breach probability functions

- The second class of security breach probability functions is given by:

$$S^{II}(z, v) = v^{\alpha z + 1}$$

- Optimal value can be found as

$$z^{II*}(v) = \frac{\ln(1 / -\alpha v L(\ln v))}{\alpha \ln v}$$

- For both functions they have shown that

$$z^*(v) < (1/e)vL.$$

v – Probability of security breach
L – Potential Loss. *vL* – Expected Loss
z – Level of Investment
S[*z*, *v*] – Revised probability of breach

Note that $1/e = 0.3679$

Propositions

- Proposition 1. For all security breach probability functions for which A1– A3 hold, there exists a loss, L , and a range of v in which increases in vulnerability result in an increase in the optimal investment in information security.
- Proposition 2. Suppose a security breach probability function meets conditions A1–A3, then it is not necessarily the case that the optimal level of investment in information security, $z^*(v)$, is weakly increasing in vulnerability, v .
- Proposition 3. Suppose the security breach probability function belongs to class I (i.e., it can be expressed as $S^I(z,v)=v/(\alpha z+1)^\beta$ for some $\alpha>0$, $\beta\geq 1$) or to class II (i.e., it can be expressed as $S^{II}(z, v) = v^{\alpha z+1}$ for some $\alpha > 0$), then $z^*(v) < (1/e) vL$. (See their Appendix for proof.)
 - The optimal investment in information security is always less than or equal to 36.79% of the loss that would be expected in the absence of any investment in security

How Can Organizations Use the Gordon-Loeb Model?

1. Estimate the potential loss (L) from a cybersecurity breach for each set of information
 - information segmentation is important.
2. Estimate the probability that an information set will be breached, by examining its vulnerability (v) to attack.
3. Create a grid with all the possible combinations of the first two steps, from low value, low vulnerability, to high value, high vulnerability.
4. Focus spending where it should reap the largest net benefits based on productivity of investments.

Recent Developments

- Widely cited in economic/financial fields.
- **Main impact:** 2017 U.S. Better Business Bureau (BBB) report recommends the Gordon-Loeb Model as "...a useful guide for organizations trying to find the right level of cybersecurity investment."
- [Cybersecurity Investment Guidance: Extensions of the Gordon and Loeb Model](#), S. Farrow, J. Szanton, 2016
- [Calibration of the Gordon-Loeb Models for the Probability of Security Breaches](#), M. Naldi, M. Flamini, 2017.
 - Values used: $v = 0.5-0.9$, $L = 1$ million, $\alpha = 4 \times 10^{-5}$, $\beta = 1$
 - Optimal about $0.2 v$
- Table based investment distribution: based on risk values of each component.

		Value of Information Sets (in \$ Million)*					
		Low	Medium		High		
Vulnerability/Threat**	Low	20	40	60	80	100	
	Medium	20%	4	8	12	16	20
		40%	8	16	24	32	40
	High	60%	12	24	36	48	60
80%		16	32	48	64	80	

*Value of Information = Potential Loss (L)

**Vulnerability/Threat = V

Low: $vL < 30$

Medium: $69 \geq vL \geq 30$

High: $vL \geq 70$

Gordon, L.A., Loeb, M.P., Zhou, L.: [Investing in cybersecurity: insights from the Gordon-Loeb model](#).
 J. Inf. Secur. 7(02), 49 (2016)

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Costs of security breaches



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Cost Models

- **Ponemon Institute**
 - Founded in 2002 by Larry Ponemon and Susan Jayson
 - conducts independent research on data protection
 - Collaborates with several large organizations and publishes annual reports
- **NetDiligence**
 - Privately-held cyber risk assessment and data breach services company.
 - Since 2001, NetDiligence has conducted thousands of enterprise-level cyber risk assessments for a broad variety of organizations
 - NetDiligence services are used by leading cyber liability insurers in the U.S. and U.K.
- **Ponemon assisted models, sponsored by**
 - Symantec (2010),
 - Megapath (2013), and
 - IBM (2014)
- **NetDiligence Model**
 - Hub International calculator (2012) and
 - contributed to the Verizon report

Cost Metrics

Total Cost of a Breach =

Incident investigation cost

+ Customer Notification/crisis management cost

+ Regulatory and industry sanctions cost

+ Class action lawsuit cost

$$\mathbf{Cost\ per\ Record} = \frac{\textit{Total cost of breach}}{\textit{number of affected records}}$$

Cost Models: Investigations

- **The Ponemon Institute and NetDiligence data/models**
 - They used proprietary data available to them.
 - They derived computational models based on their data (“calculators”).
 - Large number of factors, considerable variation in factors considered.
- **Objective of study by Algarni and Malaiya**
 - Identify the major factors that are significant
 - Build models for the factors identified.
- **Approach**
 - regenerate data using the computational engines by providing a large number of input combinations.
 - Identified and removed the factors that emerged as non-significant.
 - Developed systematic computational models.

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A consolidated approach for estimation of data security breach costs, AM Algarni, YK Malaiya
2016 2nd International Conference on Information Management (ICIM), 26-39