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Scanning Polyhedra: Fourier-Motzkin Elimination

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Outline

- Parallelizing programs with dependences
- Two levels of parallelism
  - Virtualization, virtualization, virtualization
  - Rules and when/how to break them (Volkov)
- Coarse grain wavefront parallelism
- Fine grain wavefronts
- Locality, Parallelism and Energy
- Breaking the rules – synchronization and communication of coarse grain tasks

Wavefront Parallelization

- How to parallelize computations (e.g., loops in OpenMP) that have dependences:
  - None of the loop iterations are independent

Simple examples

```
for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        A[i,j] = foo(A[i,j-1], A[i-1,j])

for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        B[j] = bar(B[j-1], B[j])

for (i=1; i<N; i++)
    for (j=1; j<M; j++)
        C[i] = baz(C[i-1], C[i])
```

Iteration Space & Data Space

- Iteration Space: set of values that the loop iterators can take
  - Rectangular region, with "corners" [1,1] and [N-1, M-1]
- Data Space: set of values of array indices accessed by the statements in the program
  - Ex 1: 2-D table, (nearly) identical to the iteration space
  - Ex 2: 1D array, bounded by [0, M-1]
  - Ex 3: 1D array, bounded by [0, N-1]

References and Dependences

- Reference: a occurrence of an array variable on either
  - left hand side (write reference)
  - right hand side (read)

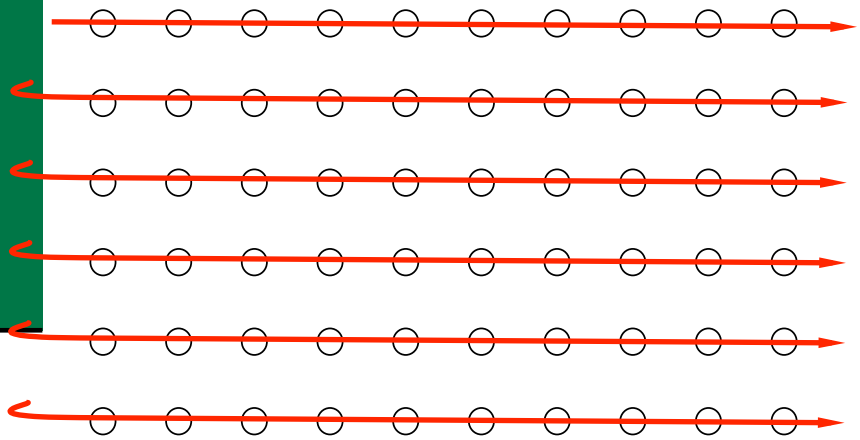
of a statement in the loop body

- Dependences: specify which iteration points depend on which others
  - can be refined if/when there are multiple statements in the program

Finding the dependences

- Very hard problem (undecidable in general) but we have simple cases
  - An iteration point [i, j] reads a memory location
  - (Many) iterations (may) have written to that location
  - Find this set (as a function of [i,j])
  - Find the "most recent writer" in this set (again, as a function of [i,j])

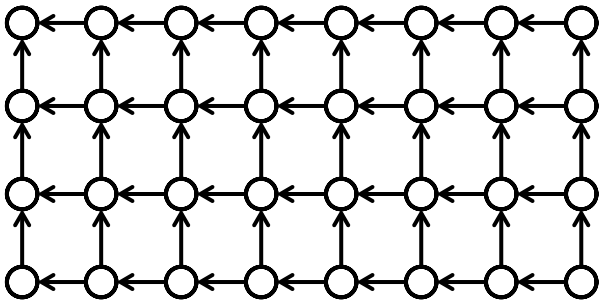
Execution order



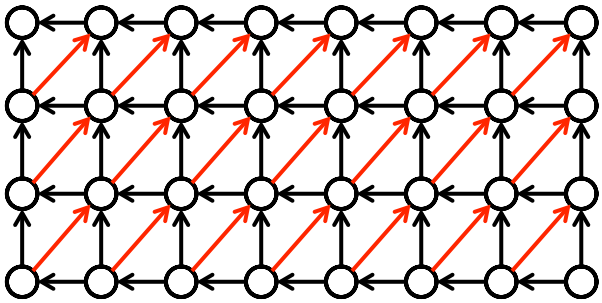
Solutions to examples

- Ex1 and Ex2 (same solution, even though the data space is very different). Iteration [i,j] depends on:
  - [i, j-1] and [i-1, j] neighbors on west and north
  - Ex2 has an additional (memory based dependence)
    - Iteration [i-1, j+1] reads a memory location that the iteration [i, j] is overwriting, that must also happen before [i, j] so it cannot be executed before its northeast neighbor
- Ex3 is more complicated
  - [i,j] depends on [i,j-1] and [i-1, M-1]

Dependence Graph (Ex 1)



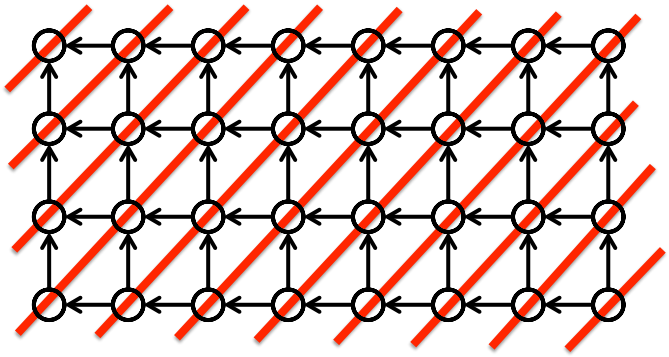
Dependence Graph (Ex 2)



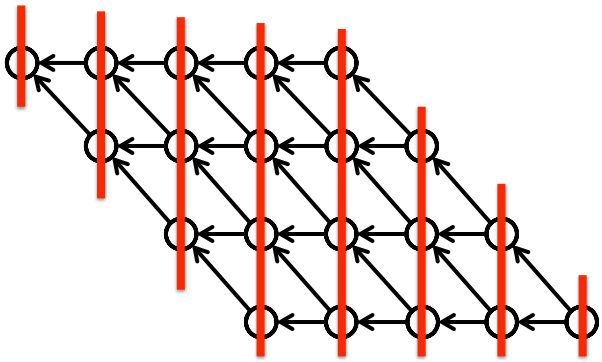
(Finally) the parallelization

- Now that we know the dependences between iterations (the dependence graph)
  - Analyze to determine what can happen at what time (hopefully many things can happen at the same time)
  - Rewrite the program to represent this new order

Ex1 wavefront parallelization



Redraw the graph



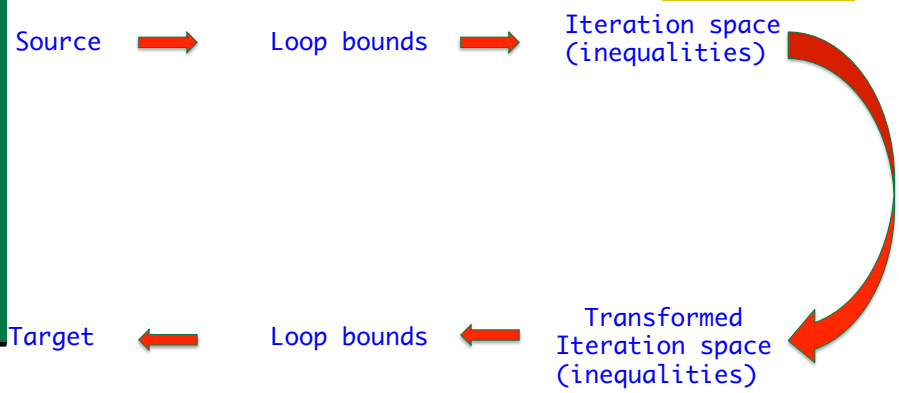
Writing the (OpenMP) code

- Node [i, j] is mapped to [p, t]
  - (i, j -> p, t) = (i, j -> i, i+j-1)
- Inverse of the transformation:
  - (p, t -> i, j) = (p, t -> p, t-p+1)
- Determine the transformed iteration space
- Write loops that traverse this
- Outer loop must be the time
- Inner loop is marked to be executed in parallel with

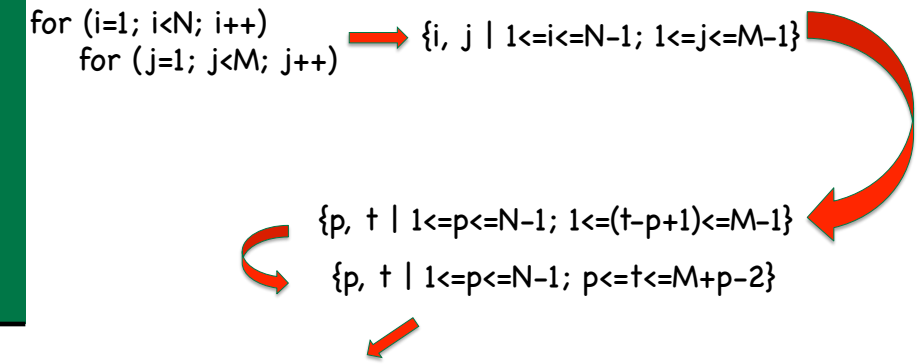
```
#pragma omp parallel for
```

- Write the new loop body

Control structure



Control structure




```
for (t=1; t<=N+M-3; t++)
    for (p=max(1,t-M+2); p<=min(t, N-1); p++) //this is parallel
```


From inequalities to loops

- Work "inside out," i.e., generate bounds on the innermost dimension (say z_n) first
- For each inequality, rearrange it into the form:
  $a_n z_n \geq exp$, for some constant coefficient a_n
  - If a_n is positive, exp/a_n is a lower bound on z_n
  - Otherwise, $-exp/a_n$ is an upper bound
- Let $l_1 \ldots l_m$ be the lower bound expressions and $u_1 \ldots u_{m'}$ be the upper bound expressions.
- The innermost loop is (with one caveat):
  $for\ (z_n = max(l_1 \ldots l_m);\ z_n < min(u_1 \ldots u_{m'});\ z_n{+}{+})$
- Recurse on the outer n-1 dimensions

Recursion: eliminate z_n

- For each pair, u_i, l_j, introduce an inequality, $u_i \geq l_j$
- Let the collection of these inequalities define the iteration space I_{n-1}
  - I_{n-1} is an (n-1)-dimensional iteration space (doesn't involve z_n)
  - The first n-1 coordinates of every point in the original iteration space, I_n also satisfy I_{n-1}
  - I_n is the intersection of I_{n-1} and loop bounds

Example (on doc cam)

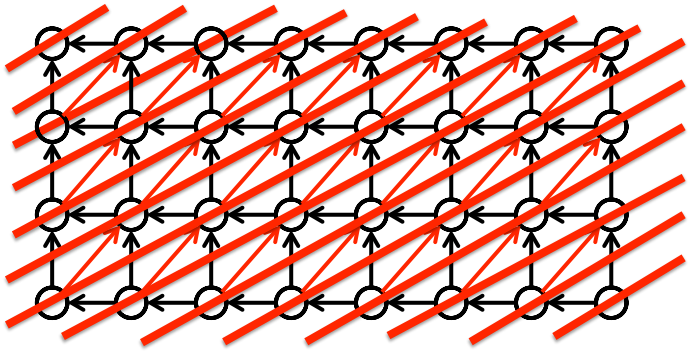
New loop body

- At each point [t, p] in the new loop,
  - Determine the original iteration point that was mapped to [t, p] (*inverse* of the rectangle-to-parallelogram transformation) Given [t, p] = [i+j-1, j] solve for [i, j] in terms of t and p.
  - Add synchronization (optional)
  - Optionally, change memory

New loop body

```
int i, j;
    for (t=1; t<=N+M-3; t++)
    #pragma omp parallel for private i, j
    for (p=max(1,t-M+2); t<=min(t,N-1); p++) {
       i = p;
       j = t-p+1;
       // insert old loop body (unchanged) here:
       // we chose t and p as brand new index names
       A[i,j] = foo(A[i,j-1], A[i-1,j]);
       }
```

Ex2 wavefront parallelization



Example 2 (contd)

- Mapping is (i, j -> p, t) = (i, j -> i, 2i+j-2)
- Inverse of the transformation:
  - (p, t -> i, j) = (p, t -> p, t-2p+2)
- Transformed iteration space:

  {p, t | 1<=p<=N-1; 1<=(t-2p+2)<=M-1}
- Rewrite as:

  {p, t | 1<=p<=N-1; t-M+3<=2p<=t+1}

Write the new loop

Example 2 (contd)

```
for (t=3; t<= 2N+M-5; t++)
    for (p = max(1, ⌈(t-M+3)/2⌉ ); p <= min( ⌊(t+1)/2⌋ , N-1) {
        //NEW LOOP BODY:
        i = p; j = t-2p+2;
        // copy the old body
        B[j]=bar(B[j], B[j-1]);
}
```


Better way

- Early preoccupation with memory:
  - Memory allocation of the original program is hurting us
- First parallelize the "full table version"
- Then make it use less memory

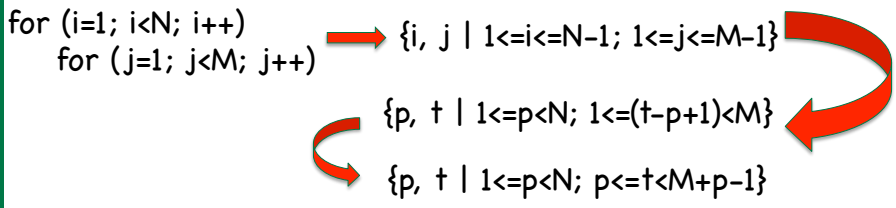
Ex1 revisited = Ex 2

```
int i, j;
    for (t=1; t<=N+M-3; t++)
    #pragma omp parallel for private i, j
    for (p=max(1,t-M+1); t<=min(t,N-1); p++) {
       i = p;
       j = t-p+1;
       // A[i,j] = foo(A[i,j-1], A[i-1,j]);
       A[i%2, j] = bar(A[i%2, j-1], A[(i-1)%2, j]);
       if (p==N-1) B[j] = A[i%2, j];
}
```


So what about CUDA

- Two levels of parallelism
  - Focus for now only on fine grain parallelism
- CUDA threads are not the same as OpenMP
  - Work done by each thread is explicitly described (parametric function of tid)
  - Implicit outer parallel loops, iterating over
    - Block coordinates, and
    - Thread coordinates
- Memory may be
  - private (registers), or
  - shared (shared memory)
- Synchronization may have to be explicit
- Tweak what we did for OpenMP to do CUDA

What's wrong with outer p?

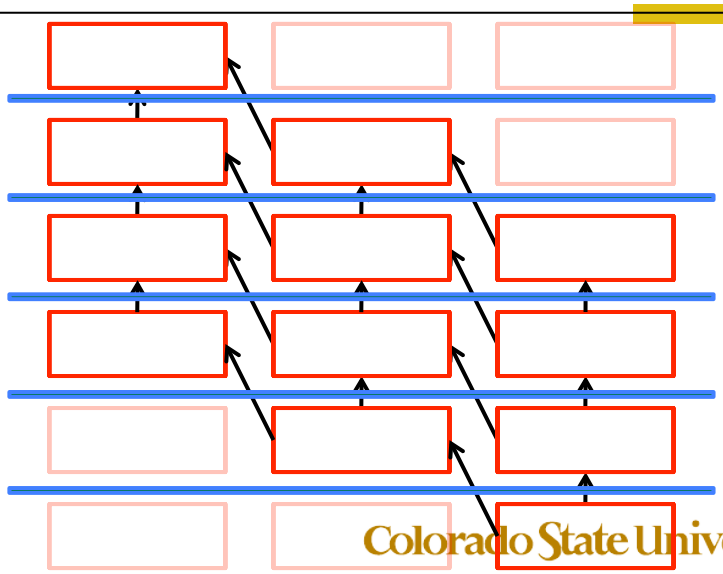



```
#pragma omp parallel for // make the outer loop parallel
for (p=1; p<=N-1; p++)
    for (t=p; t<=M+p-1; t++) {

     // body

#pragma omp barrier (what if we added this line)
    }
```


Assume numthreads = N-1

```
#pragma omp parallel for // make the outer loop parallel
for (p=1; p<=N-1; p++)
{
   for (t=0; t<p; t++) {
#pragma omp barrier
    }
    for (t=p; t<=M+p-1; t++) {// body
#pragma omp barrier
    }
   for (t=M+p; t<tmax; t++) {
#pragma omp barrier
    }
}
```

Dummies & Synchronization

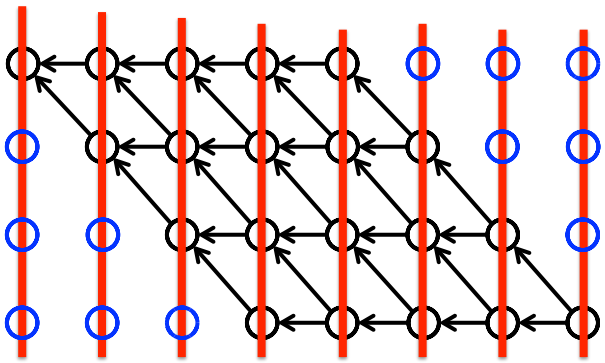


Why is outer p incorrect?

```
#pragma omp parallel for // make the outer loop parallel
for (p=1; p<=N-1; p++)
    for (t=p; t<=M+p-1; t++) {
     // body
}
```

- OpenMP semantics: all iterations of the p loop should be independent of each other
  - But iteration p
    - reads data produced by iteration p-1, and
    - produces data to be consumed by p+1
  - Solution – add explicit synchronization

Dummy (nop) nodes



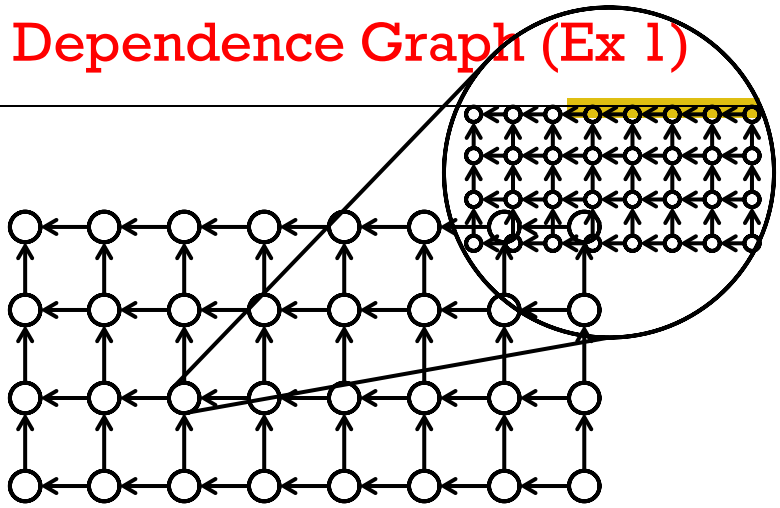
Synchronization in CUDA

- Waruna's toy kernel (array increment)
  - Each thread increments all the elements in an array (in shared memory)
  - Avoid races by taking turns
  - Thread 0 starts first and updates the first block (of width SUBTILE_WIDTH)
  - Next it moves on to the next block while thread 1 updates the one to which thread 0 just wrote
  - …

What if numthreads < N-1

- This is the more realistic case:
  - N is typically large (limited only by shared memory capacity)
  - numthreads is no more than a machine dependent upper bound (e.g., 512), usually much less than N
- Solution: divide the rows into strips of height h = (N-1)/numthreads and have each thread responsible for the whole strip
  - Avoid serialization by splitting columns into strips too
  - Basic unit is a rectangular, hxw tile

Dependence Graph (Ex 1)



Details: memory

- Each thread
  - Updates a private array of length h
    - Left/right boundary of tile
    - in registers (private to thread)
  - Communicates to next thread an array of length w
    - Top/bottom boundary of tile
    - must be in shared memory (only mechanism by which threads communicate with each other)
  - Double buffering to avoid races
  - Synchronize (and sometimes do nops)

Drawbacks and Solutions

- Latency of the pipeline fill
  - Inherent in the problem – graph has a single source
  - Other programs may have better parallelism (e.g., KPDP)
- Only uses a single SM
- Solution:
  - Do the same thing all over again
  - Tile one more level

Smth-Wtrmn code: Waruna

- Real application (biological sequence comparison)
- One twist – a diagonal dependence
- Outer level of (coarse grain parallelism)
  - Sequence of kernel calls from the host
  - Host executes the time loop iterating over the wavefronts
  - Each front has a number of independent tiles
- Inner fine grain parallelism
  - Each tile is executed by a thread block
  - In parallel (using multiple threads)
  - Use the parallelization discussed till now

Knapsack DP parallelization

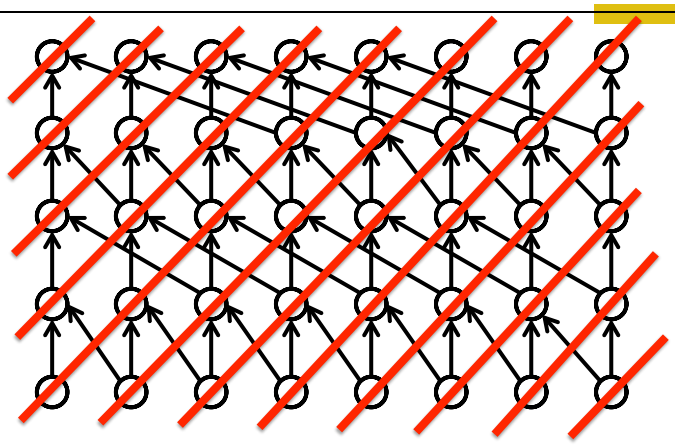
- Lessons from HW0
  - The code (i.e., function `fill-table` in HW3) was memory bound
  - omp overhead of thread spawn/sync in each of the outer n iterations
- HW3: Coarse grain parallelism
  - What is the speedup? Is it good?
  - Why?
- What happens if fine-grain parallelism is added?

Fine Grain Parallel on GPU

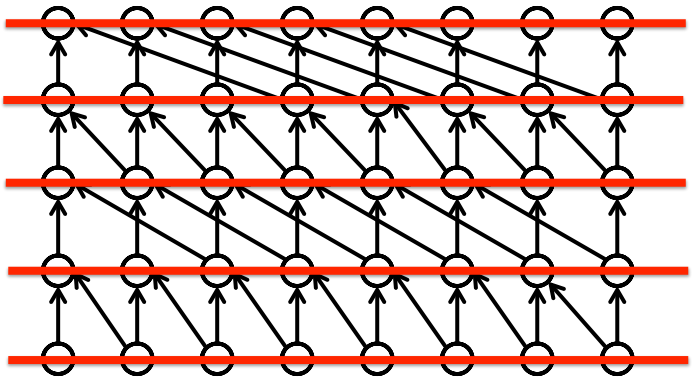
- Main challenge – make it compute bound rather than bandwidth bound
- Optimize the loop body
  - No conditionals
  - Especially conditionals using thread-id (caveat)
- Improve the balance
- Use resources on the web

http://people.maths.ox.ac.uk/gilesm/cuda/cudaconf_oxford.pdf
http://developer.download.nvidia.com/CUDA/training/cuda_webinars_dentifying_performance_limiters.pdf

Dependences of the DPKP



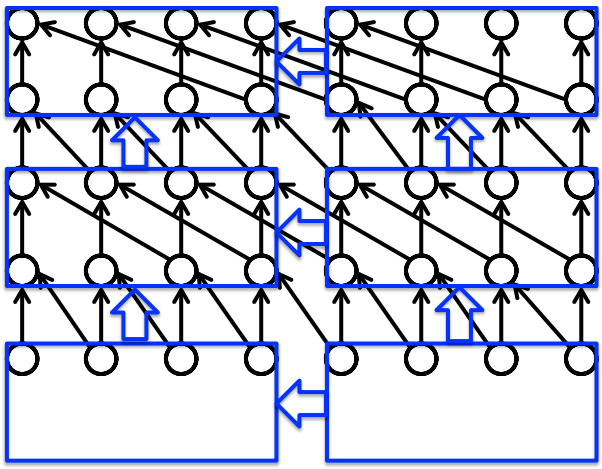
"Better" Parallelization



Pros & Cons

- Pipeline fill-flush vs "concurrent start" of all threadblocks
- Load balance
  - During fill/flush
  - In steady state: adjust stile sizes such that # threadblocks is multiple of #SMs – can be parameterized
    - Easy with "concurrent start wavefronts"
    - Complicated with [1,1] wavefront
- So do concurrent start with tiling to improve the balance

Tiled DPKP Dependences



Main Drawback

- Tiling loses the concurrent start property
- Unless
  - Tile height is 1
- Challenge – how to retain/improve balance (arithmetic intensity) with tiles of height 1
- Issue: data must be retrieved from (and restored to) global memory in each kernel call
- Solution: do a sequence of rows in a single kernel call

New issues

- There are dependences between thread-blocks
- Need for synchronization
- Data transfer via global memory read-writes
- Possibility of deadlocks?

Solution: synchronization

- Each threadblock (i.e., one designated thread, sat thread 0 in the threadblock)
  - maintains a counter indicating which row of the table it has successfully finished
  - Array if grid-size (in global memory) call it `status[]`
- A threadblock (numbered `p`)
  - does not start row number `i` until `status[p-1]` has reached `i-1`.
  - After finishing row `i`, it
    - Copies the last w[i] values of its section of the row into global memory
    - increments `status[p]` to `i`.

Deadlock?

- Previous solution relies on the runtime system's scheduling order
- What if the number of threadblocks is more than number of SMs?
- Block number p may be running and block p-1 has not even started
- Deadlock!!
- Currently runtime system schedules in the "correct order"

Deadlock: better solution

- Use atomic operations in CUDA
- Each threadblock does a "test-and-increment" to obtain a unique counter, whose value is, say ctr. Properties
  - There must be ctr-1 threadblocks that are already active (may even have terminated)
  - Therefore I will start working on the ctrth chunk of data, regardless of what my block-id is.
- Dynamic (rather than static) allocation of work to threadblocks.