High-Performance Embedded Systems-on-a-Chip

Lecture 17: Scheduling

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Limitations of Systolic Arrays

Only a (very small) proper subset of SAREs: Those that
- are Serializable,
- are Localizable,
- correspond to a Single Equation, and
- admit a One-dimensional schedule.

Question: What is beyond systolic arrays?
For each point in domain of each variable, determine:
- A time instant ⇒ schedule processor
- A place ⇒ and allocation memory
- Transform P-SARE so that indices denote either
  - time,
  - processor, or
  - memory address
- Generate code (or HDL, we hope)
Two Orthogonal Issues

- Static Analysis: what transformation to apply
  - scheduling
  - processor (& memory) allocation
- Program Transformation: manipulating the SARE
  - Rules to modify the SARE (Change of Basis)
  - Code Generation (how to interpret the transformed SARE)
Golden Rule of Static Analysis

The dependence graph cannot be explicitly constructed

- Too large
- Not (fully) known at compile time – parameters
- Explicitly constructed results are not useful

Implication: use compact information
Reduced Dependence (Multi) Graph (RDG)

- **Nodes** $\Rightarrow$ variables in the SARE
- **Edges** $\Rightarrow$ for each occurrence of $Y[f(z)]$ on the rhs of the equation for $X$, *Rightarrow* edge from $X$ to $Y$. Labeled with
  - the dependence function, $f$
  - the (sub) domain of $X$ where it occurs
  - Miscellaneous info (eg. duration, etc.)
Key Problem: Scheduling

- **Definition**: A function \( t \) such that whenever \( U[x] \) depends on \( V[y] \), then \( t(U, x) > t(V, y) \).
- **Affine schedules**:

\[
t(U, x) \equiv \lambda^U x + \alpha^U
= \lambda_1^U x_1 + \ldots + \lambda_n^U x_n + \alpha^U
\]

**Geometric interpretation**: all points executed at time \( t_0 = \lambda^U z + \alpha^U \) belong to isotemporal hyperplane with normal vector \( \lambda^U \).
Scheduling a (single) URE

\[ V[z] = \{ z \in D \} : f(V[z + d_1], \ldots, V[z + d_s] \]
Scheduling a single URE

- \( \langle \lambda, \alpha \rangle \) is valid iff for \( k = 1 \ldots s \), and \( \forall z \in D \):

\[
\lambda z + \alpha > \lambda(z + d_k) + \alpha = \lambda z + \lambda d_k
\]

i.e., \( \lambda d_k < 0 \)

- Finite number of constraints, independent of domain size. Scheduling \( \equiv \) Linear Programming

- Geometric view: Choose the hyperplanes so that dependences point backwards
Example

\[ X[i, j] = g(X[i - 1, j], X[i, j - 1]) \]

\[ t(i, j) \equiv ai + bj + \alpha \]

Schedule validity conditions

\[ [a, b][0, -1]^T < 0 \]
\[ [a, b][-1, 0]^T < 0 \]
\[ \alpha \geq 0 \]

i.e., \( \{a, b, \alpha \mid a, b > 0, \alpha \geq 0\} \)

Optimal schedule: \( t(i, j) = i + j \)
Scheduling an SURE

- Single schedule for all variables
  Not general enough: some well defined SURE’s don’t admit such a schedule (e.g. the convolution example)

- Shifted linear schedules
  Allow the $\alpha^U$ to be different for each variable, $U$, but same $\lambda$
  also not general enough

- Variable dependent schedules: different slopes for different variables)
  not general enough either

- Multidimensional schedules most general, but still
  not enough
Limits of shifted linear schedules

\[ X[i, j] = g(X[i - 1, j + 1]) \]

\[ Y[i, j] = h(Y[i + 1, j - 1]) \]

- This SURE cannot be scheduled with same-slope lines for both \( X \) and \( Y \).
- But do a simple CoB—transpose one of the vars—and now it can.
A less contrived example

\[ X[i, j] = g(X[i - 1, j + 1]) \]
\[ Y[i, j] = h(Y[i + 1, j - 1], X[i, j]) \]
\[ t_X(i, j) = a_X i + b_X j + \alpha_X \]
\[ t_Y(i, j) = a_Y i + b_Y j + \alpha_Y \]

Optimal solution

\[ t_X(i, j) = i \]
\[ t_Y(i, j) = i + 2j + 1 \]
Variable dependent schedules
Exercise

Find length of longest path reaching the green (cf. red) node at \([i, j]\)

\[
X[i, j] = f(X[i - 1, j + 1], Y[i, j - 1])
\]

\[
Y[i, j] = g(X[i, j], Y[i + 1, j - 1])
\]
• The SURE is inherently **sequential** (exercise: check it)

\[ t_X(i, j) = (i + j)(i + j + 1) + i \]
\[ = i^2 + j^2 + 2ij + 2i + j \]

\[ t_Y(i, j) = (i + j)(i + j + 1) + i + 2j + 1 \]
\[ = i^2 + j^2 + 2ij + 2i + 3j + 1 \]

• Schedule is quadratic **NOT AFFINE**

• Generalization: polynomial schedules

• Break down of geometric interpretation as **hyperplanes**
Polynomial/Multidimensional Schedules

\[ t_X[i, j] = \binom{i + j}{i} \]

\[ t_Y[i, j] = \binom{i + j}{i + 2j + 1} \]
\[ \langle \lambda, \alpha \rangle \text{ is valid iff for } k = 1 \ldots s, \text{ and } \forall z \in D \]

\[
\lambda z + \alpha > \lambda(z + d_k) + \alpha \\
= \lambda z + \lambda d_k
\]

i.e., \[ \lambda d_k < 0 \]

- Finite number of constraints, independent of domain size. \textbf{Scheduling } \equiv \textbf{Linear Programming}

\textbf{Geometric view:} Choose the hyperplanes so that dependences point \textbf{backwards}