Linearly Correlated Intermittent Failures

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Key Words—*s*-Dependence, Intermittent failures, Transient failures, Reliability analysis, *s*-Dependency in redundant modules.

Reader Aids—
Purpose: Advance the state of the art
Special math needed for explanations: Probability
Special math needed to use results: Same
Results useful to: Reliability engineers, Fault-tolerant system designers and theoreticians

Abstract—**In reliability calculation, the general assumption is to regard the behavior of redundant systems as (statistically) *s*-independent. This considerably simplifies the mathematics involved, but limits the usefulness of the results as *s*-dependence can appreciably impact the system reliability. This paper introduces two measures, using linear correlation, to describe the *s*-dependence of intermittent failures. One measure relates to the linear correlation for the entire period during which faults are active in different modules; the other measure relates to the closeness in time of the instants faults become active in different modules. Characteristics and their relationship with reliability and fault processes of these measures are considered. Irreversible processes corresponding to permanent failures are examined.**

I. INTRODUCTION

Intermittent (transient) failures [1-4] dominate the field failures in digital systems. Unlike permanent failures, intermittent failures affect the system in a complicated way. But practically all mathematical analyses begin with a series of 'reasonable' assumptions so that the analysis is mathematically tractable.

For redundant systems with intermittent failures, it has been assumed that the redundant modules behave *s*-independently [5-9]. This considerably simplifies the analysis as a single module can be examined alone, and then the cumulative effect can be studied. The assumption is, however, open to question. The modules may share the clock and the power supply, and fluctuations or noise in them can render the redundant modules susceptible to other phenomena which can cause intermittent failures. Such intermittent failures might not occur at the same time; however, they can be *s*-dependent. Other causes for *s*-dependent failures are electromagnetic radiation in the vicinity of the modules, mechanical vibration, and changes in the environment like temperature or humidity. Often, intermittent failures are caused by localized heating on IC (integrated circuit) chips; if the redundant modules are exercised by the same input sequence, the activity of such faults will be *s*-dependent.

s-Dependence is important in calculating the reliability of a redundant system. Let us consider a 2-out-of-3:G (Triple Modular Redundancy) system. If the failures in the three modules were *s*-independent, then when one module is temporarily faulty, there is a good chance that other modules will function correctly. If, however, there is some *s*-dependence, then the chances of two modules being faulty at the same time is increased, thereby increasing the chance of the output of the voter being incorrect.

In order to consider such *s*-dependence during reliability calculations, it is necessary to quantify *s*-dependence. Such a measure should relate well to the physical phenomena and should keep the analysis tractable.

This paper examines some measures of *s*-dependence between modules. After presenting a suitably defined measure, its relationship with the system parameters is investigated. When the parameters of a single module are available along with an estimated value of a *s*-dependence measure (by empirical methods), then the module parameters with *s*-dependence can be obtained and used to compute reliability. In order to obtain empirical relations for a *s*-dependence measure, multi-module experimental systems have to be studied; the fault behavior with respect to time can be recorded by using self-checking.

Notation List

FN	state in which the fault in a module is not active
FA	state in which the fault in a module is active
λ^*	Pr{transition to a *FA* in time *dt* | was in *FN*}
μ^*	Pr{transition to *FN* in time *dt* | was in *FA*}
m_u	average of r.v. *u*
m_v	average of r.v. *v*
ϱ_{uv}	linear correlation coefficient between r.v.'s *u* and *v*

Behavior of a module will be described by two states: fault not-active (*FN*) and fault active (*FA*). When a module is in state *FN*, the fault is dormant and the module operates correctly. When the module is in state *FA* the module operates incorrectly. Figure 1 models a single module [4] without reference to any other modules. By definition, the parameters λ^* and μ^* do not vary with the activity of other modules. In general, these parameters could be functions of time; for simplicity, here they are assumed to be constant.

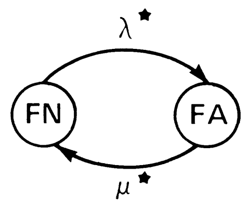

Fig. 1. Behaviour of a single module.

Measure of s-dependence: Here linear correlation will be used as a measure of *s*-dependence. The linear correlation coefficient between two r.v.'s (*u*, *v*) is:

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$$\varrho_{uv} \equiv \frac{\mathrm{Cov}\{u, v\}}{[\mathrm{Var}\{u\}\ \mathrm{Var}\{v\}]^{1/2}} = \frac{E\{(u - m_u)(v - m_v)\}}{[E\{(u - m_u)^2\}E\{(v - m_v)^2\}]^{1/2}}$$

The absolute value of ϱ_{uv} ranges between zero and one. When $\varrho_{uv} = 0$, u and v are uncorrelated. When $\varrho_{uv} = 1$, they are completely correlated. Uncorrelated r.v.'s are not necessarily *s*-independent, but no other suitable measure of *s*-dependence was found.

Correlation will be modeled in two different ways here. Generally speaking, for a certain case, one model will be more suitable than the other. The corresponding measure of correlation will be called 'duration linear correlation' and 'transition linear correlation'. The reason for this nomenclature is obvious from the definitions.

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II. DURATION LINEAR CORRELATION

The fault process in a module A is described by a random process $X_A(t)$ such that:

$$X_A(t) = \begin{cases} 0, & \text{when the module is in } FN \\ 1, & \text{when the module is in } FA \end{cases}$$

Let a similar process X_B be associated with module B. We will now examine the linear correlation between these two.

Assumptions

1. Fault processes are Markov.
2. The modules have identical distributions.
3. The ϱ_{AB} is non-negative. This means that active fault in a module does not imply less chances of the fault being active in the other.
4. The processes are ergodic, i.e. the time average is the same as the ensemble average.
5. The fault activity in one module does not cause fault activity in the other. The correlation arises because of some external events affecting both of them, i.e. a common cause.
6. The fault activity is linearly correlated in the two modules.

Notation List

$X_A(t)$, $X_B(t)$	fault processes described above
m_A	time average of $X_A(t)$
$p_{A\|B}$	$\Pr\{(X_A = 1) \mid (X_B = 1)\}$
p_A	$\Pr\{X_A = 1\}$
$\lambda, \mu, \lambda', \mu'$	parameters in figure 2
P_i	steady state probability of the system being in state i (in figure 2)

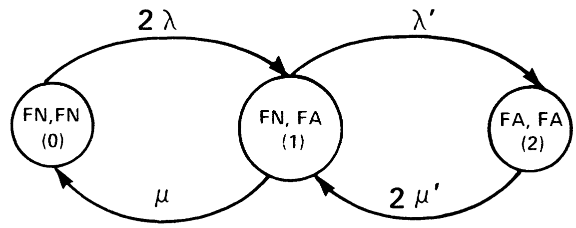

Fig. 2. Fault model for two modules with duration l.c.

By definition, duration linear correlation is:

$$\varrho_{AB}(t) = \frac{E\{(X_A - m_A)(X_B - m_B)\}}{[E\{(X_A - m_A)^2\}\ E\{(X_B - m_B)^2\}]^{1/2}}$$

As by assumption, $m_A = m_B$; and as X_A and X_B are either 0 or 1, $X_A^2 = X_A$, $X_B^2 = X_B$:

$$\varrho_{AB}(t) = [E\{X_A X_B\} - m_A^2]/[m_A - m_A^2]. \tag{1}$$

The $\varrho_{AB}(t)$ will be evaluated for steady-state behavior.

$$E\{X_A X_B\} = \sum_{i=0,1} \sum_{j=0,1} ij \Pr\{(X_A = i) \cap (X_B = j)\}$$
$$= \Pr\{(X_A = 1) \cap (X_B = 1)\}$$
$$= \Pr\{(X_A = 1) | (X_B = 1)\} \Pr\{X_B = 1\}$$

As the processes are ergodic, the ensemble averages $\Pr\{X_A = 1\}$ and $\Pr\{X_B = 1\}$ are equal to the times averages m_A and m_B. We can now rewrite (1),

$$\varrho_{AB} = (p_{A|B} - p_A)/(1 - p_A). \tag{2}$$

The ϱ_{AB} describes the steady-state behavior. If two modules A and B are not linearly correlated then $p_{A|B} = p_A$, giving $\varrho_{AB} = 0$. If the fault being active in one implies fault active in the other, then $p_{A|B} = 1$; consequently $\varrho_{AB} = 1$.

Fault activity of two modules with some linear correlation is modeled in figure 2. The parameters (λ, μ) and (λ', μ') describe the fault process in one module when the other is in *FN* and *FA* respectively. Let steady state probabilities of the system being in state 0, 1, 2 (as indicated in figure 2) be indicated by P_0, P_1, P_2. Their values can be obtained as:

$$P_0 = \mu\mu'/(\lambda\lambda' + \mu\mu' + 2\lambda\mu')$$
$$P_1 = 2\lambda\mu'/(\lambda\lambda' + \mu\mu' + 2\lambda\mu') \tag{3}$$
$$P_2 = \lambda\lambda'/(\lambda\lambda' + \mu\mu' + 2\lambda\mu')$$

As $p_{A|B} = P_2/(\tfrac{1}{2}P_1 + P_2)$ and $p_A = \tfrac{1}{2}P_1 + P_2$; we can write (2):

$$\varrho_{AB} = \frac{\mu}{\mu + \lambda} - \frac{\mu'}{\mu' + \lambda'}. \tag{4}$$

Equation (4) can be examined for the two extreme cases. The faults in the two modules are uncorrelated when $\varrho_{AB} = 0$, which requires—

$$\lambda/\mu = \lambda'/\mu'. \quad (5)$$

Condition (5) is satisfied if $\lambda = \lambda'$ and $\mu = \mu'$, which would be the case when the processes X_A and X_B are s-independent. However (5) is also satisfied if both λ' and μ' are proportional to λ and μ. In that case X_A and X_B are uncorrelated, but not s-independent.

The other extreme condition is $\varrho_{AB} = 1$, when X_A and X_B are completely correlated. This is satisfied only when—

$$\lambda/\mu = 0 \text{ and } \lambda'/\mu' \rightarrow \infty. \quad (6)$$

This is easily explained using figure 2. When $\lambda/\mu \rightarrow 0$ and $\lambda'/\mu' \rightarrow \infty$, the probability rates of entering state (*FN, FA*) are reduced while probability rates of leaving it are increased. Therefore the probability of system being in state (*FN, FA*) is reduced to zero, and thus both modules are either in state *FN* or *FA*.

In general $\varrho_{AB} \geqslant 0$, which implies—

$$\lambda'/\mu' \geqslant \lambda/\mu. \quad (7)$$

The (5), (6), (7) give the limits λ'/μ' can assume:

$$\infty \geqslant \lambda'/\mu' \geqslant \lambda/\mu. \quad (8)$$

The values of λ/μ and λ'/μ' can be obtained [13] if the unconditional parameters λ^* and μ^* (or p_A), and ϱ_{AB} are known:

$$\frac{\lambda}{\mu} = \frac{p_A(1 - \varrho_{AB})}{1 - p_A(1 - \varrho_{AB})} \quad (9a)$$

$$\frac{\lambda'}{\mu'} = \frac{p_A(1 - \varrho_{AB}) + \varrho_{AB}}{(1 - p_A)(1 - \varrho_{AB})} \quad (9b)$$

$$p_A = \lambda^*/(\lambda^* + \mu^*). \quad (10)$$

Example: Two modules A and B, have $p_A = p_B = 10^{-6}$ and $\varrho_{AB} = 10^{-2}$. Outputs of both modules are connected to a perfect voter. The two modules also provide some self-checking information to the voter, so that when one module becomes faulty, the voter can still choose the good module with a probability of 0.8. Using figure 2 and (3), the system reliability is

$$R = P_0 + 0.8P_1 = \frac{1 + 1.6(\lambda/\mu)}{(\lambda/\mu)(\lambda'/\mu') + 1 + (2\lambda/\mu)}$$

Using (9) and (10), $\lambda/\mu = 9.9 \times 10^{-7}$ and $\lambda'/\mu' = 1.01 \times 10^{-2}$. This gives $F \equiv 1 - R = 4.06 \times 10^{-7}$.

Figure 3 is a plot of F versus ϱ_{AB}. As ϱ_{AB} increases, the advantage of redundancy decreases. When ϱ_{AB} is close to unity, the reliability is essentially equal to that of a single unit.

III. TRANSITION LINEAR CORRELATION

In the previous case it was assumed that fault-activity can be linearly correlated during the entire period the fault is active in a module. This could be because of some phenomenon which in some durations tends to keep the fault active.

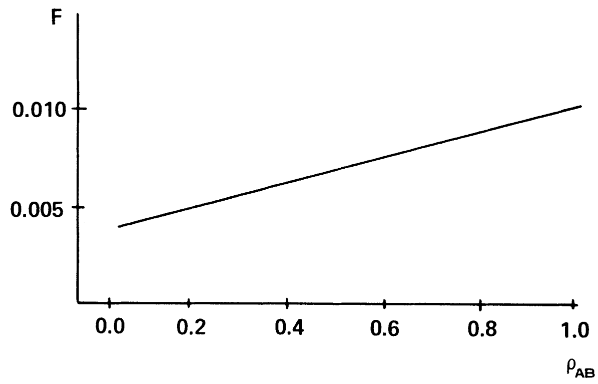

Fig. 3. Variation of unreliability F with ϱ_{AB}.

In other cases, it is possible that some phenomenon may tend to cause transition for *FN* to *FA* in the redundant modules. In that case an appropriate measure of linear correlation should relate to the probability that the two modules will have their faults become active within a short interval. Consider, for example, the fault activity of two modules as shown in figure 4. For such cases, we define a transition linear correlation, with values depending on 'arrival' of faults in two modules within a short period. To accomplish this, we define a process $Y_A(t)$, which is related to $X_A(t)$ defined in the beginning of section II (figure 5):

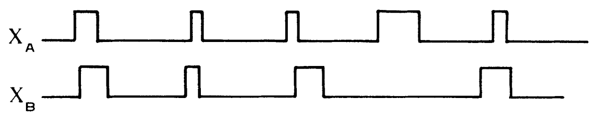

Fig. 4. Fault activity of two modules with transition l.c.

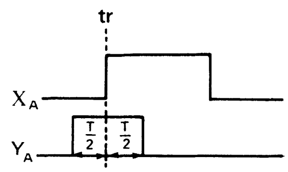

Fig. 5. Relationship between X_A and Y_A.

$$Y_A(t) = \begin{cases} 1, \text{ when } t_r - \tfrac{1}{2}T \leqslant t \leqslant t_r + \tfrac{1}{2}T \\ 0, \text{ otherwise} \end{cases} \quad (11)$$

Thus $Y_A(t) = 1$ in the neighborhood of an $FN \rightarrow FA$ transition. $Y_B(t)$ is defined similarly.

Assumptions

1. Fault processes are Markov.
2. The modules have identical distributions.
3. The processes are ergodic.
4. For two modules A and B, the $Y_A(t)$ and $Y_B(t)$ are linearly correlated.
5. The external process C is Markov. The faults may become active only during $C = 1$.

Notation List

$Y_A(t)$, $Y_B(t)$ defined above
T neighborhood defined above
C external process defined below
λ, μ parameters defining the fault process
L, M parameters defining the external process C.
P_i steady-state probability of the system being in state i (figure 6)
p'_A, p'_B $\Pr\{Y_A = 1\}$, $\Pr\{Y_B = 1\}$
$p'_{A|B}$ $\Pr\{Y_A = 1 \mid Y_B = 1\}$

Similar to the previous case, transition linear correlation can be defined as the correlation coefficient between $Y_A(t)$ and $Y_B(t)$. Considering the steady state case, we obtain:

$$\varrho_{AB} = \frac{p'_{A|B} - p'_A}{1 - p'_A}. \quad (12)$$

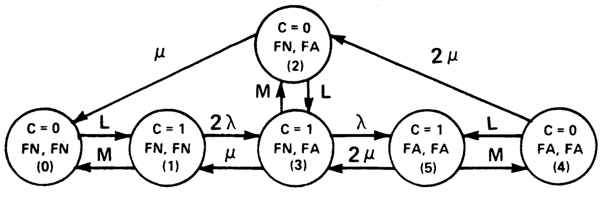

Fig. 6. Fault model for two modules with transition l.c.

We now examine (12) for a more specific case. Consider an external process C which assumes value 0 (inactive) or 1 (active) randomly. When process C is 1, it makes the correct operation of the modules susceptible to a random process, internal to each module. As a consequence, each module can be described by a random process of transition between two states FN and FA. Transitions from FN to FA are possible only when $C = 1$. However, the reverse transitions from FA to FN may occur any time. Figure 6 shows the processes in a combined form.

In order to obtain ϱ_{AB} in (12), values of $p'_{A|B}$ and p'_A have to be obtained. This can be done by obtaining the steady-state probabilities P_0 to P_5 (figure 6), and then by evaluating the appropriate integrals in the Supplement [13]. As algebraic computation becomes unwieldy, integration has to be performed numerically.

When appropriate, the following approximate relation [13] can be used:

$$\varrho_{AB} \approx \frac{\dfrac{\lambda}{M + \mu + \lambda}\,[1 - \exp[-(M + \mu + \lambda)T]] - \tfrac{1}{2}T[2\lambda P_1 + \lambda P_3]}{1 - \tfrac{1}{2}T[2\lambda P_1 + \lambda P_3]} \quad (13)$$

Figure 7 shows a plot of ϱ_{AB} against the 'neighborhood' T. For small value of T the curve is linear. For a justifiable use of the 'neighborhood' concept, T ought to be chosen in this region.

IV. MEASURE OF LINEAR CORRELATION FOR IRREVERSIBLE PROCESSES:

For processes which describe the generation of permanent failures, steady-state probabilities do not exist. However, a useful correlation coefficient can still be defined.

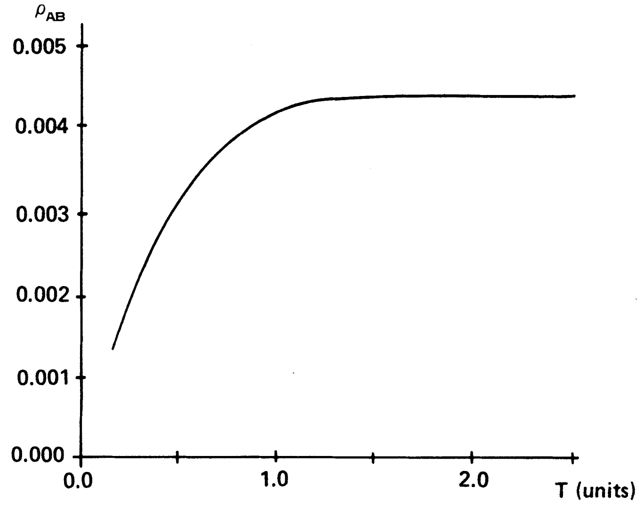

Fig. 7. Transition l.c. vs defined neighborhood.

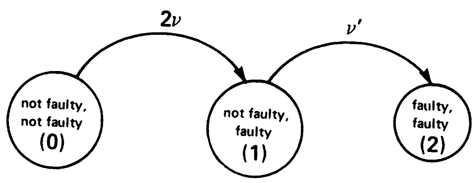

Fig. 8. Model for two modules with permanent faults.

A. *Duration Linear Correlation*

Figure 8 shows a model equivalent to figure 2. Again two processes X_A and X_B are defined such that:

$$X_A = \begin{cases} 0, & \text{if fault in A is not present} \\ 1, & \text{if fault in } A \text{ is present} \end{cases}$$

Assumptions

1. The fault processes are Markov.
2. The modules have identical distribution.
3. The fault processes are linearly correlated.

Notation List

X_A, X_B processes defined above
ν, ν' hazard rates in figure 8.
P_i $\Pr\{$system is in state i (figure 8)$\}$

The ϱ_{AB} now is a function of time:

$$\varrho_{AB} = \frac{P_2 - (P_2 + \frac{1}{2}P_1)^2}{(P_2 + \frac{1}{2}P_1) - (P_2 + \frac{1}{2}P_1)^2} \tag{14}$$

$$P_1 \equiv \frac{2\nu}{2\nu - \nu'} (\exp[-\nu' t] - \exp[-2\nu t]) \tag{15a}$$

$$P_2 \equiv 1 - \frac{2\nu\nu'}{2\nu - \nu'} \left[\frac{\exp[-\nu' t]}{\nu'} - \frac{\exp[-2\nu t]}{2\nu} \right]. \tag{15b}$$

It can be seen from (14)-(15) that as $t \to \infty$, we have $\varrho_{AB} \to 1$.

B. *Transition Linear Correlation*

For permanent failures, the transition linear correlation might be a more suitable measure, as it will relate to nearly simultaneous failures under the influence of a randomly occuring external event. Figure 6 would still be valid if $\mu = 0$. However, an approximate relation like (13) would not be valid since steady-state probabilities do not exist. Numerical simulation however can be effectively used.

V. EXTENSIONS

It has been assumed above the transition rates are constant. The resulting exponential distribution is memoryless, hence such systems are Markov. It has been determined that in some cases Weibull distribution has a closer fit [11, 12]. Results have been extended for this case, and can be found in the Supplement [13].

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